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A NOTE ON PROPERTIES OF TEMPORAL INTUITIONISTIC FUZZY SETS OF SECOND TYPE

R. Srinivasan* & K. Rajesh**
* Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Tamilnadu
** Full Time Research Scholar, Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi, Tamilnadu

Abstract:

Fuzzy Sets are generalized from the notion of classical (crisp) sets (Zadeh1965) [6]. A fuzzy subset A of X can be characterized with a membership function μ_A: X → [0, 1]. Krassimir T. Atanassov further generalized fuzzy sets into Intuitionistic Fuzzy Sets (IFSs) in which non- membership function ν_A: X → [0, 1] is also considered [1]. Further he extended IFSs into Temporal Intuitionistic Fuzzy Sets (TIFSs) in which time- moments are also taken into consideration. TIFSs are useful in time-based mathematical modeling. In this paper, the present authors further introduce the extension of Intuitionistic Fuzzy Sets namely Temporal Intuitionistic Fuzzy Sets of Second Type and study some of their properties.

Key Words: Fuzzy sets (FS), Intuitionistic Fuzzy Sets (IFS), Intuitionistic Fuzzy Sets of Second Type (IFSST), Temporal Intuitionistic Fuzzy Sets (TIFS) & Temporal Intuitionistic Fuzzy Sets of Second Type (TIFSST).

1. Introduction:

Fuzzy Sets (FSs) are used in various aspects of science and technology such as Engineering and medicine. The theory of Fuzzy set is one of the most important invention of time. Time is an important feature in real world. The IFSs are useful to deal with vagueness of knowledge.

Temporal Intuitionistic Fuzzy sets[1] is defined by Atanassov in 1991.In his definition, membership and non-membership degree of an element change with both of the element and time moment. This is one of the most important extensions of IFS. Because, our real world situations are generally spatio - temporal. Thus, by the theory of TIFS, real world situations like weather, medicine, economy, image-video processing…can be handled more realistic and effective. It is well-known that time is monotone and time is a fundamental issue for modeling dynamic information. The rest of the paper is designed as follows: In Section 2, we give some basic definitions. In Section 3, we define TIFSST and study their some properties. This paper is concluded in section 4.

2. Preliminaries:

In this section, we give some basic definitions.

Definition 2.1 [6]

Let X be a non- empty set. A Fuzzy Set A in X is defined as an object of the form

$$A = \{< x, \mu_A(x) > | \; x \in X\},$$

Where $\mu_A : X \rightarrow [0,1]$ is the membership function of the fuzzy set A.

Definition 2.2 [1]

An intuitionistic fuzzy set (IFS) A in X is defined as an object of the following form.

$$A = \{< x, \; \mu_A(x), \; \nu_A(x) > | \; x \in X\},$$

Where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ defines the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

The ordinary fuzzy set may be written as the following form

$$A = \{< x,\ \mu_A(x),\ 1 - \mu_A(x) > | x \in X\}.$$

That is all the fuzzy sets are IFSs.

Definition 2.3 [1]

Let a set X be fixed.An intuitionistic fuzzy set of second type (IFSST) A in X is defined as an object of the following form.

$$A = \{< x,\ \mu_A(x),\ \nu_A(x) > | \ x \in X\}$$

Where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ defines the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq {\mu_A}^2(x) + {\nu_A}^2(x) \leq 1$$

Remark:

It's obvious that for all real numbers a,b∈[0, 1], if $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, then $0 \leq {\mu_A}^2(x) + {\nu_A}^2(x) \leq 1$. Hence all the IFSs are IFSST.

Definition 2.4 [1]

Let X be an universe and T be a non-empty set and the elements of t are time moments. A Temporal Intuitionistic Fuzzy Set (TIFS) is an object of the form.

$$A(T) = \{< x,\ \mu_A(x,t),\ \nu_A(x,t) > | < x,t > \in X \times T\},$$

Where

1. $A \subset X$ is a fixed set.
2. $\mu_A(x,\ t) + \nu_A(x,\ t) \leq 1$ For every $< x,t > \in X \times T$.
3. $\mu_A(x,t)$ and $\nu_A(x,t)$ are the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$ at the time-moment $t \in T$.

Let X be an universe and T be a non-empty set and the elements of T are time-moments. A Temporal Intuitionistic Fuzzy Set (TIFS) is an object of the following form

$$A(T) = \{< x, \mu_A(x,t), \nu_A(x,t) > | < x,t > \in X \times T\}$$

3. Temporal Intuitionistic Fuzzy Sets of Second Type:

In this section, we define the Temporal Intuitionistic Fuzzy Sets of Second Type which is an extension of TIFS and some basic operations

Definition 3.1:

We define a temporal intuitionistic fuzzy set of second type (TIFSST) as an following object

$$A(T) = \{< x,\ \mu_A(x,t),\ \nu_A(x,t) > | < x,t > \in X \times T\},$$

Where

1. $A \subset X$ is a fixed set.
2. ${\mu_A}^2(x) + {\nu_A}^2(x) \leq 1$ For every $< x,t > \in X \times T$.
3. $\mu_A(x,t)$ and $\nu_A(x,t)$ are the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$ at the time-moment $t \in T$.

Note:

1. Obviously, every ordinary IFS can be regarded as TIFSST for which T is a singleton set.
2. All operations and operators on the IFSs can be defined for the TIFSST.

3.1 Basic Operators on TIFSST:

Consider two TIFSSTs, $A(T') = \{< (x,t), \mu_A(x,t), \nu_A(x,t) > | < x,t > \in X \times T'\}$ and $B(T'') = \{< (x,t), \mu_B(x,t), \nu_B(x,t) > | < x,t > \in X \times T''\}$

Definition 3.2:

We define the following basic operations on two TIFSSTs, $A(T')$ and $B(T'')$.

1. $A(T') \cap B(T'') = \{< (x,t), \min(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)), max(\bar{\nu}_A(x,t), \bar{\nu}_B(x,t)) > \mid < x,t > \in X \times (T' \times T'')\}$
2. $A(T') \cup B(T'') = \{< (x,t), \max(\bar{\mu}_A(x,t), \bar{\mu}_B(x,t)), min(\bar{\nu}_A(x,t), \bar{\nu}_B(x,t)) > \mid < x,t > \in X \times (T' \times T'')\}$
3. $\bar{A}(T') = \{< (x,t), \bar{\nu}_A(x,t), \bar{\mu}_A(x,t) > \mid < x,t > \in X \times T'\}$,

Where

$$\bar{\mu}_A(x,t) = \begin{cases} \mu_A if t\epsilon T' \\ 0 \quad if t\epsilon T'' - T' \end{cases}, \bar{\nu}_A(x,t) = \begin{cases} \nu_A if t\epsilon T' \\ 1 \quad if t\epsilon T'' - T' \end{cases}$$

$$\bar{\mu}_B(x,t) = \begin{cases} \mu_B if t\epsilon T'' \\ 0 \quad if t\epsilon T' - T'' \end{cases}, \bar{\nu}_B(x,t) = \begin{cases} \nu_B if t\epsilon T'' \\ 1 \quad if t\epsilon T' - T'' \end{cases}$$

Definition 3.3:

We define the following two operators C* and I* over TIFSST A(T)

$$C^*(A(T)) = \{< (x,t), \max_{t\epsilon T} \mu_{A(T)}(x,t), \min_{t\epsilon T} \nu_{A(T)}(x,t) > \mid (x,t) \epsilon X\}$$

$$I^*(A(T)) = \{< (x,t), \min_{t\epsilon T} \mu_{A(T)}(x,t), \max_{t\epsilon T} \nu_{A(T)}(x,t) > \mid (x,t) \epsilon X\}$$

Theorem 3.1:

For every TIFSST A(T), C*(A(T))and I*(A(T))are TIFSSTs

Proof:

Let $\max_{t\epsilon T} \mu_{A(T)}(x,t) = \mu_{A(T)}(x,t')$ for some $t' \in T$ and $\min_{t\epsilon T} \mu_{A(T)}(x,t) = \mu_{A(T)}(x,t'')$ for some $t'' \in T$.

Therefore, $\nu_{A(T)}(x,t'') \leq \nu_{A(T)}(x,t')$ and

$$\begin{aligned} \max_{t\epsilon T} \mu_{A(T)}(x,t) + \min_{t\epsilon T} \nu_{A(T)}(x,t) &= \mu^2{}_{A(T)}(x,t') + \nu^2{}_{A(T)}(x,t'') \\ &\leq \mu^2{}_{A(T)}(x,t') + \nu^2{}_{A(T)}(x,t') \\ &\leq 1 \end{aligned}$$

Hence, $C^*(A(T))$ is a TIFSST.

Similarly, we prove $I^*(A(T))$ is a TIFSST.

4. Properties of Temporal Intuitionistic Fuzzy Sets of Second Type:

In this section, we establish some properties of Temporal Intuitionistic Fuzzy Sets of Second Type.

Theorem 4.1:

For every TIFSST A(T) the following relations between C*(A(T)) and I*(A(T)) hold.

(i) C*(C*(A(T))) = C*(A(T))
(ii) C*(I*(A(T))) = I*(A(T))
(iii) I*(C*(A(T))) = C*(A(T))
(iv) I*(I*(A(T))) = I*(A(T))

Proof:

(i) $C^*(C^*(A(T))) = C^*C^* \{< (x,t), \mu_A(x,t), \nu_A(x,t) > \mid < x,t > \in X \times T\}$

$$= C^* \{< (x,t), \max_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T} \nu_{A(T)}(x,t) > \mid (x,t) \epsilon X \times T\}$$

$$= \{< (x,t), \max_{t\in T} \max_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T} \min_{t\in T} \nu_{A(T)}(x,t) > \mid (x,t) \epsilon X \times T\}$$

$$= \{< (x,t), \max_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T} \nu_{A(T)}(x,t) > \mid (x,t) \epsilon X \times T\}$$

$$= C^*(A(T))$$

(ii) $C^*(I^*(A(T))) = C^*(I^* \{< (x,t), \mu_A(x,t), \nu_A(x,t) > \mid < x,t > \in X \times T\})$

$= C^*\left\{< (x,t),\ \min_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\epsilon X\times T\right\}$

$= \left\{< (x,t), \max_{t\in T}\min_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T}\max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\epsilon X\times T\right\}$

$= \left\{< (x,t), \min_{t\in T}\max_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T}\min_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\epsilon X\times T\right\}$

$= \left\{< (x,t), \min_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\epsilon X\times T\right\}$

$= I^*(A(T))$

(iii) $I^*(C^*(A(T))) = I^*\left(C^*\{< (x,t), \mu_A(x,t), \nu_A(x,t) > \mid < x,t > \in X\times T'\}\right)$

$= I^*\left\{< (x,t),\ \max_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \min_{t\in T}\max_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T}\min_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \max_{t\in T}\min_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T}\max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \max_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= C^*(A(T))$

(iv) $I^*(I^*(A(T))) = I^*\left(I^*\{< (x,t), \mu_A(x,t), \nu_A(x,t) > \mid < x,t > \in X\times T'\}\right)$

$= I^*\left\{< (x,t),\ \min_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \min_{t\in T}\min_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T}\max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \min_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= I^*(A(T))$

Theorem 4.2:

For every TIFSSTs A(T) the following relations between C*(A) and I*(A) hold.

(i) $C(C^*(A(T))) = C^*(C(A(T)))$

(ii) $I(I^*(A(T))) = I^*(I(A(T)))$

Proof:

(i) $C(C^*(A(T))) = C\,C^*\{< (x,t), \mu_A(x,t), \nu_A(x,t) > \mid < x,t > \in X\times T\}$

$= C\left\{< (x,t),\ \max_{t\in T} \mu_{A(T)}(x,t), \min_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t),\ \max \max_{t\in T} \mu_{A(T)}(x,t), \min \min_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \max_{t\in T} \max \mu_{A(T)}(x,t), \min_{t\in T} \min \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= C^*(C(A(T)))$

(ii) $I(I^*(A(T))) = I\left(I^*\{< (x,t), \mu_A(x,t), \nu_A(x,t) > \mid < x,t > \in X\times T'\}\right)$

$= I\left\{< (x,t),\ \min_{t\in T} \mu_{A(T)}(x,t), \max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t),\ \min \min_{t\in T} \mu_{A(T)}(x,t), \max \max_{t\in T} \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= \left\{< (x,t), \min_{t\in T} \min \mu_{A(T)}(x,t), \max_{t\in T} \max \nu_{A(T)}(x,t) > \middle| (x,t)\ \epsilon(X\times T)\right\}$

$= I^*(I(A(T)))$

Theorem 4.3:

For every two Temporal Intuitionistic Fuzzy Sets of Second Type $A(T')$ and $B(T'')$ the following are true, if $T' \cap T'' = \emptyset$

(i) $C^*(A(T') \cap B(T'')) = C^*(A(T')) \cap C^*(B(T''))$

(ii) $C^*(A(T') \cup B(T'')) = C^*(A(T')) \cup C^*(B(T''))$

Proof:

Given $T' \cap T'' = \emptyset$ Let $T = T' \cup T''$

Consider, $C^*(A(T') \cap B(T''))$

$= C^*\{< (x,t), \min(\mu_A(x,t), 0), \max(\nu_A(x,t), 1) > | (x,t) \in X \times T\}$ if $t \in T'$

$= \{< (x,t), 0,1 > | (x,t) \in X \times T)\}$ if $t \in T'$

and $C^*(A(T')) \cap C^*(B(T''))$

$$= \{< (x,t), \min\left(\max_{t \in T} \bar{\mu}_A, (x,t), 1\right), \max\left(\min_{t \in T} \bar{\nu}_A, (x,t), 1\right) > | (x,t) \in X \times T\}$$

if $t \in T'$

$= \{< (x,t), 0,1 > | (x,t) \in X \times T)\}$ if $t \in T'$

It is analogous to prove that $C^*(A(T') \cap B(T'')) = C^*(A(T')) \cap C^*(B(T''))$ if $t \in T''$

Similarly,

$$C^*(A(T') \cup B(T'')) = C^*\{< (x,t), \max(\mu_A(x,t), \mu_B(x,t)), \min(\nu_A(x,t), \nu_B(x,t)) > | < x,t > \in X \times T)\}$$

$$= \left\{< (x,t), \max_{t \in T} \max(\mu_A(x,t), \mu_B(x,t), 1), \min_{t \in T} \min(\nu_A(x,t), \nu_B(x,t), 0) > | < x,t > \in X \times T\right\}$$

$$= \{< (x,t), 1, 0 > | < x,t > \in X \times T\}$$

and

$$C^*(A(T')) = \left\{< (x,t), \max_{t \in T} \mu_{A(T')}(x,t), \min_{t \in T} \nu_{A(T')}(x,t) > | < x,t > \in X \times T'\right\}$$

$$C^*(B(T'')) = \left\{< (x,t), \max_{t \in T} \mu_{B(T'')}(x,t), \min_{t \in T} \nu_{B(T'')}(x,t) > | < x,t > \in X \times T''\right\}$$

$C^*(A(T') \cup B(T'')) = \{< (x,t), 1, 0 > | < x,t > \in X \times T\}$

So, $C^*(A(T') \cup B(T'')) = C^*(A(T')) \cup C^*(B(T''))$

Theorem 4.4:

For every TIFSSTs A(T), then $\overline{I^*(\overline{A}(T))} = C^*(A(T))$

Proof:

By the definition

$$\overline{A}(T) = \{< (x,t), \nu_A(x,t), \mu_A(x,t) > | < x,t > \in X \times T\}$$

$$\overline{I^*(\overline{A(T)})} = \overline{I^*\{< (x,t), \nu_A(x,t), \mu_A(x,t) > | < x,t > \in X \times T\}}$$

$$= \overline{\{< (x,t), \min_{t \in T} \nu_{A(T)}(x,t), \max_{t \in T} \mu_{A(T)}(x,t) > | (x,t) \epsilon (X \times T)\}}$$

$$= \left\{< (x,t), \max_{t \in T} \mu_{A(T)}(x,t), \min_{t \in T} \nu_{A(T)}(x,t) > | (x,t) \epsilon (X \times T)\right\}$$

$$= C^*(A(T))$$

5. Conclusion:

We have made an attempt to prove some properties of TIFSST. It is still open to check whether there exist a TIFSST in case of the operators already defined on an IFS and TIFS.

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