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A STUDY ON PROPERTIES OF INTUITIONISTIC FUZZY SETS OF THIRD TYPE

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Abstract:

In this paper, we study some of the properties of some operators on Intuitionistic Fuzzy Sets of Third Type.

Key Words: Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy Set of Second Type (IFSST), Intuitionistic Fuzzy Set of Root Type (IFSRT), Intuitionistic Fuzzy Set of Third Type (IFSTT).

1. Introduction:

To overcome the uncertainty and vagueness, the inherent in the real world, L. A. Zadeh introduced the notion of fuzzy sets in 1965. In 1983, K. T. Atanassov introduced the extended notion of Intuitionistic Fuzzy Sets. The authors further extended the Intuitionistic Fuzzy Sets namely, Intuitionistic Fuzzy Sets of Third Type and studied some of their properties. In this paper, we have studied some of the properties of some operators on Intuitionistic Fuzzy Sets of Third Type. In section 2, we recollect some basic definitions and in section 3, we have proved some properties of operators D_α, $F_{\alpha,\beta}$, $G_{\alpha,\beta}$, $H_{\alpha,\beta}$ and $H^*_{\alpha,\beta}$. The paper is concluded in section 4.

2. Preliminaries:

In this section, we give some definitions of IFS and its extensions and operators on its extensions.

Definition 2.1: [1] Let X be a non-empty set. An *IFS* A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$for each $x \in X$.

Definition 2.2: [1] Let X be a non-empty set. An *IFSST* A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and $0 \le \mu_A^2(x) + \nu_A^2(x) \le 1$ for each $x \in X$.

Definition 2.3: [2] Let X be the non-empty set. An *IFSRT* A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and $0 \le \frac{\sqrt{\mu_A(x)}}{2} + \frac{\sqrt{\nu_A(x)}}{2} \le 1$ for each $x \in X$.

Definition 2.4: [3] Let X be the non-empty set. An *IFSTT* A in X is defined as an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

Where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the membership and non-membership functions of A, respectively, and $0 \le \mu_A^3(x) + \nu_A^3(x) \le 1$ for each $x \in X$.

Definition 2.5: [3] Let A and B be two *IFSTT*s of the non-empty set X such that

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\},$$

We define the following operations on A and B:

(i) $A \subset B$ iff $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$, for all $x \in X$
(ii) $A \supset B$ iff $\mu_A(x) \ge \mu_B(x)$ and $\nu_A(x) \le \nu_B(x)$, for all $x \in X$
(iii) $A = B$ iff $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$, for all $x \in X$
(iv) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
(v) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
(vi) $\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$.

Definition 2.6: [3] The degree of non-determinacy (uncertainty) of an element $x \in X$ in the *IFSTT* A is defined by

$$\pi_A(x) = \sqrt[3]{1 - \mu_A^3(x) - \nu_A^3(x)}$$

Remark: In case of ordinary fuzzy sets, $\pi_A(x) = 0$, for every $x \in X$.

Definition 2.7: [4] For every *IFSTT* A, we define the following two topological operators:

$$C(A) = \{\langle x, K, L\rangle : x \in X\},$$

Where

$$K = \max_{y \in X} \mu_A(y)$$
$$L = \min_{y \in X} \nu_A(y)$$

And

$$I(A) = \{\langle x, k, l\rangle : x \in X\},$$

Where

$$k = \min_{y \in X} \mu_A(y)$$
$$l = \max_{y \in X} \nu_A(y)$$

We call these operators a "Closure" and "Interior" of A over the universe X respectively.

Definition 2.8: [5] Let $\alpha \in [0,1]$ be a fixed number. Given an *IFSTT* A, an operator D_α on A, is defined as

$$D_\alpha(A) = \left\{\langle x, \sqrt[3]{\mu_A^3(x) + \alpha\, \pi_A^3(x)}, \sqrt[3]{\nu_A^3(x) + (1-\alpha)\, \pi_A^3(x)}\rangle : x \in X\right\}$$

Remark: It is easy to prove that $D_\alpha(A)$ is an *IFSTT.*

Definition 2.9: [5] Let $\alpha, \beta \in [0,1]$ and $\alpha + \beta \le 1$. The operator $F_{\alpha,\beta}$ on an *IFSTT* A, is defined as

$$F_{\alpha,\beta}(A) = \left\{\langle x, \sqrt[3]{\mu_A^3(x) + \alpha\, \pi_A^3(x)}, \sqrt[3]{\nu_A^3(x) + \beta\, \pi_A^3(x)}\rangle : x \in X\right\}$$

Remark: It is obvious that $F_{\alpha,\beta}(A)$ is an *IFSTT.*

Definition 2.10: [6] Let $\alpha, \beta \in [0,1]$. Given an *IFSTT* A, an operator $G_{\alpha,\beta}$ on A is defined as

$$G_{\alpha,\beta}(A) = \{\langle x, \sqrt[3]{\alpha}\, \mu_A(x), \sqrt[3]{\beta}\, \nu_A(x)\rangle : x \in X\}$$

Remark: It is clear that, $G_{\alpha,\beta}(A)$ is an *IFSTT.*

Definition 2.11: [6] Let $\alpha, \beta \in [0,1]$. Given an *IFSTT* A, an operator $H_{\alpha,\beta}$ on A is defined as

$$H_{\alpha,\beta}(A) = \left\{\langle x, \sqrt[3]{\alpha}\, \mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta\, \pi_A^3(x)}\rangle : x \in X\right\}$$

Remark: Clearly, $H_{\alpha,\beta}(A)$ is an *IFSTT.*

Definition 2.12: [6] Let $\alpha, \beta \in [0,1]$. Given an *IFSTT* A, an operator $H^*_{\alpha,\beta}$ on A is defined as

$$H^*_{\alpha,\beta}(A) = \left\{\langle x, \sqrt[3]{\alpha}\, \mu_A(x), \sqrt[3]{\nu_A^3(x) + \beta(1 - \alpha\, \mu_A^3(x) - \nu_A^3(x))}\rangle : x \in X\right\}$$

Remark: Clearly, $H^*_{\alpha,\beta}(A)$ is an *IFSTT.*

3. Properties of Some Operators on IFSTT:

Proposition 3.1: For every two real numbers $\alpha, \beta \in [0,1]$ and for every *IFSTT* A, we have

(i) $D_\alpha(C(A)) \supset C(D_\alpha(A))$
(ii) $D_\alpha(I(A)) \subset I(D_\alpha(A))$
(iii) $G_{\alpha,\beta}(C(A)) = C\big(G_{\alpha,\beta}(A)\big)$
(iv) $G_{\alpha,\beta}(I(A)) = I\big(G_{\alpha,\beta}(A)\big)$

Proof:

(i)$D_\alpha\big(C(A)\big) = D_\alpha\left(\left\{\langle x, \max_{y\in X} \mu_A(y), \min_{y\in X} \nu_A(y)\rangle : x \in X\right\}\right)$

$$=\left\{\langle x, \sqrt[3]{\max_{y\in X} \mu_A^3(y) + \alpha\, \pi_A^3(y)}, \sqrt[3]{\min_{y\in X} \nu_A^3(y) + (1-\alpha)\, \pi_A^3(y)}\rangle : x \in X\right\}$$

$$=\left\{\langle x, \sqrt[3]{\max_{y\in X} \mu_A^3(y) + \alpha(1 - \max_{y\in X} \mu_A^3(y) - \min_{y\in X} \nu_A^3(y))}, \sqrt[3]{\min_{y\in X} \nu_A^3(y) + (1-\alpha)(1 - \max_{y\in X} \mu_A^3(y) - \min_{y\in X} \nu_A^3(y))}\rangle : x \in X\right\}$$

$$\supset \left\{\langle x, \sqrt[3]{\max_{y\in X} \mu_A^3(y) + \alpha(1 - \max_{y\in X} \mu_A^3(y) - \max_{y\in X} \nu_A^3(y))}, \sqrt[3]{\min_{y\in X} \nu_A^3(y) + (1-\alpha)(1 - \min_{y\in X} \mu_A^3(y) - \min_{y\in X} \nu_A^3(y))}\rangle : x \in X\right\}$$

$$=\left\{\langle x, \sqrt[3]{\max_{y\in X} (\mu_A^3(y) + \alpha(1 - \mu_A^3(y) - \nu_A^3(y)))}, \sqrt[3]{\min_{y\in X} (\nu_A^3(y) + (1-\alpha)(1 - \mu_A^3(y) - \nu_A^3(y)))}\rangle : x \in X\right\}$$

$$=\left\{\langle x, \sqrt[3]{\max_{y\in X} (\mu_A^3(y) + \alpha\, \pi_A^3(y))}, \sqrt[3]{\min_{y\in X} (\nu_A^3(y) + (1-\alpha)\, \pi_A^3(y))}\rangle : x \in X\right\}$$

$$=\left\{\langle x, \max_{y\in X}\left(\sqrt[3]{\mu_A^3(y) + \alpha\, \pi_A^3(y)}\right), \min_{y\in X}\left(\sqrt[3]{\nu_A^3(y) + (1-\alpha)\, \pi_A^3(y)}\right)\rangle : x \in X\right\}$$

$$=C\left(\left\{\langle x, \sqrt[3]{\mu_A^3(x) + \alpha\, \pi_A^3(x)}, \sqrt[3]{\nu_A^3(x) + (1-\alpha)\, \pi_A^3(x)}\rangle : x \in X\right\}\right)$$

$$= C(D_\alpha(A))$$

(ii)$D_\alpha\big(I(A)\big) = D_\alpha\left(\left\{\langle x, \min_{y\in X} \mu_A(y), \max_{y\in X} \nu_A(y)\rangle : x \in X\right\}\right)$

$$=\left\{\langle x, \sqrt[3]{\min_{y\in X} \mu_A^3(y) + \alpha\, \pi_A^3(y)}, \sqrt[3]{\max_{y\in X} \nu_A^3(y) + (1-\alpha)\, \pi_A^3(y)}\rangle : x \in X\right\}$$

$$=\left\{\langle x, \sqrt[3]{\min_{y\in X} \mu_A^3(y) + \alpha(1 - \min_{y\in X} \mu_A^3(y) - \max_{y\in X} \nu_A^3(y))}, \sqrt[3]{\max_{y\in X} \nu_A^3(y) + (1-\alpha)(1 - \min_{y\in X} \mu_A^3(y) - \max_{y\in X} \nu_A^3(y))}\rangle : x \in X\right\}$$

$$\subset \left\{\langle x, \sqrt[3]{\min_{y\in X} \mu_A^3(y) + \alpha(1 - \min_{y\in X} \mu_A^3(y) - \min_{y\in X} \nu_A^3(y))},\right.$$

$$\sqrt[3]{\max_{y\in X} \nu_A^3(y) + (1-\alpha)(1-\max_{y\in X}\mu_A^3(y) - \max_{y\in X}\nu_A^3(y))}\,\rangle : x\in X\Big\}$$

$$=\Big\{\langle x, \sqrt[3]{\min_{y\in X}(\mu_A^3(y)+\alpha(1-\mu_A^3(y)-\nu_A^3(y)))},$$

$$\sqrt[3]{\max_{y\in X}(\nu_A^3(y)+(1-\alpha)(1-\mu_A^3(y)-\nu_A^3(y)))}\,\rangle : x\in X\Big\}$$

$$=\Big\{\langle x, \sqrt[3]{\min_{y\in X}(\mu_A^3(y)+\alpha\,\pi_A^3(y))}, \sqrt[3]{\max_{y\in X}(\nu_A^3(y)+(1-\alpha)\,\pi_A^3(y))}\,\rangle : x\in X\Big\}$$

$$=\Big\{\langle x, \min_{y\in X}\left(\sqrt[3]{\mu_A^3(y)+\alpha\,\pi_A^3(y)}\right), \max_{y\in X}\left(\sqrt[3]{\nu_A^3(y)+(1-\alpha)\,\pi_A^3(y)}\right)\rangle : x\in X\Big\}$$

$$= I\left(\Big\{\langle x, \sqrt[3]{\mu_A^3(x)+\alpha\,\pi_A^3(x)}, \sqrt[3]{\nu_A^3(x)+(1-\alpha)\,\pi_A^3(x)}\rangle : x\in X\Big\}\right)$$

$$= I(D_\alpha(A))$$

(iii) $G_{\alpha,\beta}(C(A)) = G_{\alpha,\beta}\left(\Big\{\langle x, \max_{y\in X}\mu_A(y), \min_{y\in X}\nu_A(y)\rangle : x\in X\Big\}\right)$

$$=\Big\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\beta}\min_{y\in X}\nu_A(y)\rangle : x\in X\Big\}$$

$$=\Big\{\langle x, \max_{y\in X}(\sqrt[3]{\alpha}\,\mu_A(y)), \min_{y\in X}(\sqrt[3]{\beta}\,\nu_A(y))\rangle : x\in X\Big\}$$

$$= C\left(\Big\{\langle x, \sqrt[3]{\alpha}\,\mu_A(y), \sqrt[3]{\beta}\,\nu_A(y) : x\in X\Big\}\right)$$

$$= C\left(G_{\alpha,\beta}(A)\right)$$

(iv) $G_{\alpha,\beta}(I(A)) = G_{\alpha,\beta}\left(\Big\{\langle x, \min_{y\in X}\mu_A(y), \max_{y\in X}\nu_A(y)\rangle : x\in X\Big\}\right)$

$$=\Big\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\beta}\max_{y\in X}\nu_A(y) : x\in X\Big\}$$

$$=\Big\{\langle x, \min_{y\in X}(\sqrt[3]{\alpha}\mu_A(y)), \max_{y\in X}(\sqrt[3]{\beta}\nu_A(y)) : x\in X\Big\}$$

$$= I\left(\Big\{\langle x, \sqrt[3]{\alpha}\mu_A(y), \sqrt[3]{\beta}\nu_A(y) : x\in X\Big\}\right)$$

$$= I\left(G_{\alpha,\beta}(A)\right)$$

This completes the proof.

Proposition 3.2: For every two real numbers $\alpha,\beta \in [0,1]$ and for every *IFSTT* A, we have

- (i) $H_{\alpha,\beta}(C(A)) \supset C(H_{\alpha,\beta}(A))$
- (ii) $H_{\alpha,\beta}(I(A)) \subset I(H_{\alpha,\beta}(A))$
- (iii) $H^*_{\alpha,\beta}(C(A)) \supset C(H^*_{\alpha,\beta}(A))$
- (iv) $H^*_{\alpha,\beta}(I(A)) \subset I(H^*_{\alpha,\beta}(A))$

Proof:

(i) $H_{\alpha,\beta}(C(A)) = H_{\alpha,\beta}\left(\Big\{\langle x, \max_{y\in X}\mu_A(y), \min_{y\in X}\nu_A(y)\rangle : x\in X\Big\}\right)$

$$=\Big\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}\nu_A^3(y)+\beta\,\pi_A^3(y)}\rangle : x\in X\Big\}$$

$$=\Big\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}\nu_A^3(y)+\beta(1-\max_{y\in X}\mu_A^3(y)-\min_{y\in X}\nu_A^3(y))}\rangle : x\in X\Big\}$$

$$\supset\Big\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}\nu_A^3(y)+\beta(1-\min_{y\in X}\mu_A^3(y)-\min_{y\in X}\nu_A^3(y))}\rangle : x\in X\Big\}$$

$$=\left\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}(\nu_A^3(y)+\beta(1-\mu_A^3(y)-\nu_A^3(y)))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \max_{y\in X}\sqrt[3]{\alpha}\,\mu_A(y), \min_{y\in X}\sqrt[3]{\nu_A^3(y)+\beta(1-\mu_A^3(y)-\nu_A^3(y))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \max_{y\in X}\sqrt[3]{\alpha}\,\mu_A(y), \min_{y\in X}\sqrt[3]{\nu_A^3(y)+\beta\,\pi_A^3(y)}\rangle : x\in X\right\}$$
$$=C\left(\left\{\langle x, \sqrt[3]{\alpha}\,\mu_A(x), \sqrt[3]{\nu_A^3(x)+\beta\,\pi_A^3(x)}\rangle : x\in X\right\}\right)$$
$$= C(H_{\alpha,\beta}(A))$$

(ii) $H_{\alpha,\beta}(I(A)) = H_{\alpha,\beta}\left(\left\{\langle x, \min_{y\in X}\mu_A(y), \max_{y\in X}\nu_A(y)\rangle : x\in X\right\}\right)$

$$=\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}\nu_A^3(y)+\beta\,\pi_A^3(y)}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}\nu_A^3(y)+\beta(1-\min_{y\in X}\mu_A^3(y)-\max_{y\in X}\nu_A^3(y))}\rangle : x\in X\right\}$$
$$\subset\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}\nu_A^3(y)+\beta(1-\max_{y\in X}\mu_A^3(y)-\max_{y\in X}\nu_A^3(y))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}(\nu_A^3(y)+\beta(1-\mu_A^3(y)-\nu_A^3(y)))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \min_{y\in X}\sqrt[3]{\alpha}\,\mu_A(y), \max_{y\in X}\sqrt[3]{\nu_A^3(y)+\beta(1-\mu_A^3(y)-\nu_A^3(y))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \min_{y\in X}\sqrt[3]{\alpha}\,\mu_A(y), \max_{y\in X}\sqrt[3]{\nu_A^3(y)+\beta\,\pi_A^3(y)}\rangle : x\in X\right\}$$
$$= I\left(\left\{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(x)+\beta\,\pi_A^3(x)}\rangle : x\in X\right\}\right)$$
$$= I(H_{\alpha,\beta}(A))$$

(iii) $H^*_{\alpha,\beta}(C(A)) = H^*_{\alpha,\beta}\left(\left\{\langle x, \max_{y\in X}\mu_A(y), \min_{y\in X}\nu_A(y)\rangle : x\in X\right\}\right)$

$$=\left\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}\nu_A^3(x)+\beta(1-\alpha\max_{y\in X}\mu_A^3(x)-\min_{y\in X}\nu_A^3(x))}\rangle : x\in X\right\}$$
$$\supset\left\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}\nu_A^3(x)+\beta(1-\alpha\min_{y\in X}\mu_A^3(x)-\min_{y\in X}\nu_A^3(x))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \sqrt[3]{\alpha}\max_{y\in X}\mu_A(y), \sqrt[3]{\min_{y\in X}(\nu_A^3(y)+\beta(1-\alpha\mu_A^3(y)-\nu_A^3(y)))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \max_{y\in X}\sqrt[3]{\alpha}\,\mu_A(y), \min_{y\in X}\sqrt[3]{\nu_A^3(y)+\beta(1-\alpha\mu_A^3(y)-\nu_A^3(y))}\rangle : x\in X\right\}$$
$$=C\left(\left\{\langle x, \sqrt[3]{\alpha}\,\mu_A(x), \sqrt[3]{\nu_A^3(y)+\beta(1-\alpha\,\mu_A^3(y)-\nu_A^3(y))}\rangle : x\in X\right\}\right)$$
$$= C(H^*_{\alpha,\beta}(A))$$

(iv) $H^*_{\alpha,\beta}(I(A)) = H^*_{\alpha,\beta}\left(\left\{\langle x, \min_{y\in X}\mu_A(y), \max_{y\in X}\nu_A(y)\rangle : x\in X\right\}\right)$

$$=\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}\nu_A^3(x)+\beta(1-\alpha\min_{y\in X}\mu_A^3(x)-\max_{y\in X}\nu_A^3(x))}\rangle : x\in X\right\}$$
$$\subset\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}\nu_A^3(x)+\beta(1-\alpha\max_{y\in X}\mu_A^3(x)-\max_{y\in X}\nu_A^3(x))}\rangle : x\in X\right\}$$
$$=\left\{\langle x, \sqrt[3]{\alpha}\min_{y\in X}\mu_A(y), \sqrt[3]{\max_{y\in X}(\nu_A^3(y)+\beta(1-\alpha\,\mu_A^3(y)-\nu_A^3(y)))}\rangle : x\in X\right\}$$

$=\left\{\langle x, \min_{y\in X} \sqrt[3]{\alpha}\mu_A(y), \max_{y\in X} \sqrt[3]{\nu_A^3(y) + \beta(1-\alpha\mu_A^3(y) - \nu_A^3(y))}\rangle : x \in X\right\}$

$= I\left(\left\{\langle x, \sqrt[3]{\alpha}\mu_A(x), \sqrt[3]{\nu_A^3(y) + \beta(1-\alpha\,\mu_A^3(y) - \nu_A^3(y))}\rangle : x \in X\right\}\right)$

$= I(H^*_{\alpha,\beta}(A))$

This completes the proof.

Theorem 3.1: For every *IFSTT* A and all real numbers $\alpha, \beta, \gamma, \delta \in [0,1]$ such that $\alpha + \beta \leq 1$ and $\gamma + \delta \leq 1$, $F_{\alpha,\beta}(F_{\gamma,\delta}(A)) = F_{\alpha+\gamma-\alpha.\gamma-\alpha.\delta,\beta+\delta-\beta.\gamma-\beta.\delta}(A)$

Proof:

$F_{\alpha,\beta}(F_{\gamma,\delta}(A)) = F_{\alpha,\beta}\left(\left\{\langle x, \sqrt[3]{\mu_A^3(x) + \gamma\,\pi_A^3(x)}, \sqrt[3]{\nu_A^3(x) + \delta\,\pi_A^3(x)}\rangle : x \in X\right\}\right)$

$= \left\{\langle x, \sqrt[3]{\mu_A^3(x) + \gamma\,\pi_A^3(x) + \alpha(1 - \mu_A^3(x) - \gamma\pi_A^3(x) - \nu_A^3(x) - \delta\pi_A^3(x))},\right.$

$\left.\sqrt[3]{\nu_A^3(x) + \delta\,\pi_A^3(x) + \beta(1 - \mu_A^3(x) - \gamma\pi_A^3(x) - \nu_A^3(x) - \delta\pi_A^3(x))} : x \in X\right\}$

$= \left\{\langle x, \sqrt[3]{\mu_A^3(x) + \gamma\,\pi_A^3(x) + \alpha\pi_A^3(x) + \pi_A^3(x)(-\alpha.\gamma - \alpha.\delta)},\right.$

$\left.\sqrt[3]{\nu_A^3(x) + \delta\,\pi_A^3(x) + \beta\pi_A^3(x) + \pi_A^3(x)(-\beta.\gamma - \beta.\delta)} : x \in X\right\}$

$= \left\{\langle x, \sqrt[3]{\mu_A^3(x) + (\alpha + \gamma - \alpha.\gamma - \alpha.\delta)\,\pi_A^3(x)},\right.$

$\left.\sqrt[3]{\nu_A^3(x) + (\beta + \delta - \beta.\gamma - \beta.\delta)\,\pi_A^3(x)} : x \in X\right\}$

$= F_{\alpha+\gamma-\alpha.\gamma-\alpha.\delta,\beta+\delta-\beta.\gamma-\beta.\delta}(A)$

This proves the theorem.

4. Conclusion:

In this paper, we have established some properties of some operators on *IFSTT*. It is still open to check the operators already defined on *IFS* in case of *IFSTT.*

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