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SOME CHARACTERIZATIONS ON OPERATIONS IN SOFT SETS

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Abstract:

In this paper we have defined symmetric difference, absorption, associative of soft sets and discussed some of their properties.

Index Terms: Soft Set, Symmetric Difference, Absorption & Associative Properties

1. Introduction:

Soft set theory has received much attention, since its introduction by Molodtsov [4]. The concept and basic properties of soft set theory and presented in [3, 4]. Soft set theory has been applied in many fields. Maji *et. al* [3] worked on some new operations in soft set theory. Onyeozii and Gwary [5] studied fundamentals of soft set theory. Chinnadurai *et. al* [1,2] discussed interval valued fuzzy soft gamma semigroups and intuitionistic fuzzy soft Gamma semigroups. In this paper, some characterizations on operations in soft sets viz., symmetric difference, absorption and associative properties are discussed

2. Preliminaries:

Definition: 2.1

Let U be an initial universe set and E be a set of parameters. Let P(U)denotes the power set of U and A⊂E. A pair (F,A) is called s soft set over U, where F is a mapping given by, $F:A\rightarrow P(U)$.

Definition: 2.2

Union of two soft sets of (F,A) and (G,B) over the common universe U is the soft set (H,C), where $C=A\cup B$ and $\forall e\in C$,

$$\begin{aligned} H(e) &= F(e) && if \quad e\in A-B \\ &= G(e) && if \quad e\in B-A \\ &= F(e)\cup G(e) && if \quad e\in A\cap B. \end{aligned}$$

We write, $(F,A)\tilde{\cup}(G,B)=(H,C)$.

Definition: 2.3

Intersection of two soft sets of (F,A) and (G,B) over the common universe U is the soft set (H,C), where $C=A\cap B$ and $\forall e\in C$,

$$\begin{aligned} H(e) &= F(e) && if \quad e\in A-B \\ &= G(e) && if \quad e\in B-A \\ &= F(e)\cap G(e) && if \quad e\in A\cap B. \end{aligned}$$

We write, $(F,A)\tilde{\cap}(G,B)=(H,C)$.

Definition: 2.4

The difference of two soft sets of (F,A) and (G,B) over the common universe U is the soft set (H,C), where $C=A\cap B$ and $\forall e\in C$,

$$H(e) = F(e) - G(e)$$

We write, $(F,A)-(G,B)=(H,C.)$

Definition: 2.5

Symmetric difference of two soft sets of (F,A) and (G,B) over the common universe U is the soft set denoted as $(F,A)\tilde{\Delta}(G,B)$ and its defined by,

$$(F,A)\tilde{\Delta}(G,B)=((F,A)-(G,B))\tilde{\cup}((G,B)-(F,A)).$$

Note: 2.6

$(i)\alpha \notin F(e)\cup G(e)$ iff $\alpha \notin F(e)$ and $\alpha \notin G(e)$

$(i)\alpha \notin F(e)\cap G(e)$ iff $\alpha \notin F(e)$ or $\alpha \notin G(e)$

$(i)\alpha \notin F(e)-G(e)$ iff $\alpha \notin F(e)$ or $\alpha \notin G(e)$

3. Main Results:

Theorem: 3.1

For any three soft sets (F,A) (G,B) and (H,C) we have,

(i) $(F,A)-(G,B)=(F,A)-((F,A)\tilde{\cap}(G,B))=((F,A)\tilde{\cup}(G,B))-(G,B)$

$(ii)(F,A)-((G,B)-(H,C))=((F,A)-(G,B))\tilde{\cup}((F,A)\tilde{\cap}(H,C)).$

Proof:

(i) Let, $(F,A)-(G,B),$

Assume that, $\alpha \in F(e)-G(e),$ then $\alpha \in F(e)$ and $\alpha \notin G(e).$

Now, $\alpha \notin G(e) \Rightarrow \alpha \notin F(e)\cap G(e)$

$\Rightarrow \alpha \in F(e)$ and $\alpha \notin (F(e)\cap G(e)).$

$\Rightarrow \alpha \in F(e) - (F(e)\cap G(e)).$

Then, $(F,A)-((F,A)\tilde{\cap}(G,B)).$

Therefore, $(F,A)-(G,B)\subseteq (F,A)-((F,A)\tilde{\cap}(G,B)).$ $\cdots\cdots\cdots(1)$

Now, let $(F,A)-((F,A)\tilde{\cap}(G,B)),$

$\Rightarrow \alpha \in F(e) - (F(e)\cap G(e)).$

Then, $\alpha \in F(e)$ and $\alpha \notin F(e)\cap G(e)$

$\Rightarrow \alpha \in F(e)$ and $(\alpha \notin F(e)\, or\, \alpha \notin G(e)).$

$\Rightarrow \alpha \in F(e)$ and $\alpha \notin G(e).$

$\Rightarrow \alpha \in F(e) - G(e)$

$\Rightarrow (F,A)-(G,B).$

Therefore, $(F,A)-((F,A)\tilde{\cap}(G,B))\subseteq (F,A)-(G,B).$ $\cdots\cdots\cdots(2)$

From (1) and (2), we get,

$(F,A)-(G,B)=(F,A)-((F,A)\tilde{\cap}(G,B))$

(ii) Let, $(F,A)-((G,B)-(H,C))$

Assume that, $\alpha \in F(e) - (G(e)-H(e)).$

Then, $\alpha \in F(e)$ and $\alpha \notin (G(e)-H(e))$

$\Rightarrow \alpha \in F(e)$ and $(\alpha \notin G(e) or\, \alpha \notin H(e)).$

$\Rightarrow (\alpha \in F(e)$ and $\alpha \notin G(e))$ or $(\alpha \in F(e)$ and $\alpha \notin H(e)).$

$\Rightarrow \alpha \in (F(e)-G(e))\cup (F(e)\cap H(e)).$

$\Rightarrow ((F,A)-(G,B))\tilde{\cup}((F,A)\tilde{\cap}(H,C)).$

Hence,

$(F,A)-((G,B)-(H,C)) \subseteq ((F,A)-(G,B)) \tilde{\cup} ((F,A) \tilde{\cap} (H,C)).$ ......(3)

Similarly, we get,

$((F,A)-(G,B)) \tilde{\cup} ((F,A) \tilde{\cap} (H,C)) \subseteq (F,A)-((G,B)-(H,C))$ ......(4)

From (3) and (4), we get,

$(F,A)-((G,B)-(H,C)) = ((F,A)-(G,B)) \tilde{\cup} ((F,A) \tilde{\cap} (H,C)).$

Hence proved.

Theorem: 3.2

For any three soft sets (F,A) (G,B) and (H,C) then,

(i) $(F,A)-[(G,B) \tilde{\cup} (H,C)] = [(F,A)-(G,B)] \tilde{\cap} [(F,A)-(H,C)]$

(ii) $(F,A)-[(G,B) \tilde{\cap} (H,C)] = [(F,A)-(G,B)] \tilde{\cup} [(F,A)-(H,C)]$

Proof:

(i) $(F,A)-[(G,B) \tilde{\cup} (H,C)] = [(F,A)-(G,B)] \tilde{\cap} [(F,A)-(H,C)]$

$(G,B) \tilde{\cup} (H,C) = (K,D)$ where, $D = B \cup C$, $\forall e \in C$,

$$K(e) = G(e) \quad if \quad e \in B-C$$
$$= H(e) \quad if \quad e \in C-B$$
$$= G(e) \cup H(e) \quad if \quad e \in B \cap C.$$

Let, $(F,A)-(K,D) = F(e)-[G(e)-H(e)]$

$\Rightarrow \alpha \in [F(e)-(G(e) \cup H(e))]$

$\Rightarrow \alpha \in F(e)$ and $\alpha \notin [G(e) \cup H(e)]$

$\Rightarrow \alpha \in F(e)$ and $\alpha \notin G(e)$ and $\alpha \notin H(e)$

$\Rightarrow \alpha \in [F(e)-G(e)]$ and $\alpha \in [F(e)-H(e)]$

$\Rightarrow \alpha \in [F(e)-G(e)] \cap [F(e)-H(e)]$

Hence,

$(F,A)-[(G,B) \tilde{\cup} (H,C)] \subseteq [(F,A)-(G,B)] \tilde{\cap} [(F,A)-(H,C)]$ ......(1)

Similarly, we get,

$[(F,A)-(G,B)] \tilde{\cap} [(F,A)-(H,C)] \subseteq (F,A)-[(G,B) \tilde{\cup} (H,C)]$ ......(2)

From (1) and (2), we get,

$(F,A)-[(G,B) \tilde{\cup} (H,C)] = [(F,A)-(G,B)] \tilde{\cap} [(F,A)-(H,C)]$

(ii) $(F,A)-[(G,B) \tilde{\cap} (H,C)] = [(F,A)-(G,B)] \tilde{\cup} [(F,A)-(H,C)]$

$(G,B) \tilde{\cap} (H,C) = (K,D)$ where, $D = B \cap C$, $\forall e \in C$,

$$K(e) = G(e) \quad if \quad e \in B-C$$
$$= H(e) \quad if \quad e \in C-B$$
$$= G(e) \cap H(e) \quad if \quad e \in B \cap C.$$

Let, $(F,A)-(K,D) = F(e)-[G(e)-H(e)]$

$\Rightarrow \alpha \in [F(e)-(G(e) \cap H(e))]$

$\Rightarrow \alpha \in F(e)$ and $\alpha \notin [G(e) \cap H(e)]$

$\Rightarrow \alpha \in F(e)$ and $\alpha \notin G(e)$ or $\alpha \notin H(e)$

$\Rightarrow \alpha \in [F(e)-G(e)]$ or $\alpha \in [F(e)-H(e)]$

$\Rightarrow \alpha \in [F(e)-G(e)] \cup [F(e)-H(e)]$

Hence,

$$(F,A)-[(G,B)\tilde{\cap}(H,C)]\subseteq[(F,A)-(G,B)]\tilde{\cup}[(F,A)-(H,C)] \qquad \cdots\cdots(3)$$

Similarly, we get,

$$[(F,A)-(G,B)]\tilde{\cup}[(F,A)-(H,C)]\subseteq(F,A)-[(G,B)\tilde{\cap}(H,C)] \qquad \cdots\cdots(4)$$

From (3) and (4) we get,

$$(F,A)-[(G,B)\tilde{\cap}(H,C)]=[(F,A)-(G,B)]\tilde{\cup}[(F,A)-(H,C)].$$

Theorem: 3.3

For any three soft sets (F,A) (G,B) then,

(i) $(F,A)\tilde{\Delta}(G,B)=[(F,A)\tilde{\cup}(G,B)]-[(F,A)\tilde{\cap}(G,B)]$

(ii) $(F,A)\tilde{\Delta}(G,B)=(G,B)\tilde{\Delta}(F,A).$

Proof:

(i) Let, $(F,A)\tilde{\Delta}(G,B)$

$\Rightarrow[(F,A)-(G,B)]\tilde{\cup}[(G,B)-(F,A)]$

$\Rightarrow\alpha\in[F(e)-G(e)]\cup[G(e)-F(e)]$

$\Rightarrow\alpha\in[F(e)-G(e)]$ or $\alpha\in[G(e)-F(e)]$

$\Rightarrow[\alpha\in F(e)\ and\ \alpha\notin G(e)]$ or $[\alpha\in G(e)\ and\ \alpha\notin F(e)]$

$\Rightarrow[\alpha\in F(e)\ or\ \alpha\in G(e)]$ and $[\alpha\notin G(e)\ and\ \alpha\notin F(e)]$

$\Rightarrow\alpha\in[F(e)\cup G(e)]$ and $\alpha\notin[F(e)\cap G(e)]$

$\Rightarrow\alpha\in[(F(e)\cup G(e)-F(e)\cap G(e))]$

$$(F,A)\tilde{\Delta}(G,B)\subseteq[(F,A)\tilde{\cup}(G,B)]-[(F,A)\tilde{\cap}(G,B)] \qquad \cdots\cdots(1)$$

Similarly, we get,

$$[(F,A)\tilde{\cup}(G,B)]-[(F,A)\tilde{\cap}(G,B)]\subseteq(F,A)\tilde{\Delta}(G,B)\cdots \qquad \cdots\cdots(2)$$

From (3) and (4) we get,

$$(F,A)\tilde{\Delta}(G,B)=[(F,A)\tilde{\cup}(G,B)]-[(F,A)\tilde{\cap}(G,B)]$$

$$\begin{aligned}(ii)\ (F,A)\tilde{\Delta}(G,B)&=((F,A)-(G,B))\tilde{\cup}((G,B)-(F,A))\\&=(F(e)-G(e))\cup(G(e)-F(e))\\&=(G(e)-F(e))\cup(F(e)-G(e))\\&=((G,B)-(F,A))\tilde{\cup}((F,A)-(G,B))\\&=(G,B)\tilde{\Delta}(F,A)\end{aligned}$$

Hence proved.

Theorem: 3.4

For any three soft sets (F,A) (G,B) then,

(i) $(F,A)\tilde{\cup}((F,A)\tilde{\cap}(G,B))=(F,A)$

(ii) $(F,A)\tilde{\cap}((F,A)\tilde{\cup}(G,B))=(F,A)$

Proof:

(i) Let, $(F,A)\tilde{\cap}(G,B)=(H,C)$

By the definition,

$$H(e)=F(e)\cap G(e) \qquad if \quad e\in A\cap B$$

$$(F,A)\tilde{\cup}((F,A)\tilde{\cap}(G,B))=(F,A)\tilde{\cup}(H,C)=(K,D)$$

By the definition,

$$K(e) = F(e) \quad \text{if} \quad e \in A - C$$
$$= H(e) \quad \text{if} \quad e \in C - A$$
$$= F(e) \cup H(e) \quad \text{if} \quad e \in A \cap C$$
$$= F(e) \cup (F(e) \cap G(e))$$
$$= F(e)$$

Then, we get, $K(e) = F(e)$

Hence,

$(F, A) \tilde{\cup} ((F, A) \tilde{\cap} (G, B)) = (F, A)$

(i) Let, $(F, A) \tilde{\cap} (G, B) = (H, C)$

By the definition,

$$H(e) = F(e) \quad \text{if} \quad e \in A - B$$
$$= G(e) \quad \text{if} \quad e \in B - A$$
$$= F(e) \cup G(e) \quad \text{if} \quad e \in A \cap B$$

$(F, A) \tilde{\cap} ((F, A) \tilde{\cup} (G, B)) = (F, A) \tilde{\cap} (H, C) = (K, D)$

By the definition,

$$H(e) = F(e) \cap H(e) \quad \text{if} \quad e \in A \cap C$$
$$= F(e) \cap (F(e) \cup G(e))$$
$$= F(e)$$

Then, we get, $K(e) = F(e)$

Hence,

$(F, A) \tilde{\cap} ((F, A) \tilde{\cup} (G, B)) = (F, A)$

Hence Proved

Theorem: 3.5

If (F, A) (G, B) and (H, C) are three soft sets over the universal set U, then

(i) $(F, A) \tilde{\cup} ((G, B) \tilde{\cup} (H, C)) = ((F, A) \tilde{\cup} (G, B)) \tilde{\cup} (H, C) = (F, A) \tilde{\cup} (G, B) \tilde{\cup} (H, C)$

(ii) $(F, A) \tilde{\cap} ((G, B) \tilde{\cap} (H, C)) = ((F, A) \tilde{\cap} (G, B)) \tilde{\cap} (H, C) = (F, A) \tilde{\cap} (G, B) \tilde{\cap} (H, C)$

Proof:

(i) Suppose that, $(G, B) \tilde{\cup} (H, C) = (I, D)$ where $D = B \cup C$ and $\forall e \in D$,

$$I(e) = G(e) \quad \text{if} \quad e \in B - C$$
$$= H(e) \quad \text{if} \quad e \in C - B$$
$$= G(e) \cup H(e) \quad \text{if} \quad e \in B \cap C$$

Since $(F, A) \tilde{\cup} ((G, B) \tilde{\cup} (H, C)) = (F, A) \tilde{\cup} (I, D)$, we suppose that,

$(F, A) \tilde{\cup} (I, D) = (J, M)$, where, $M = A \cup D = A \cup B \cup C$ and $\forall e \in M$,

$$J(e) = G(e) \quad \text{if} \quad e \in B - C - A$$
$$= H(e) \quad \text{if} \quad e \in C - B - A$$
$$= F(e) \quad \text{if} \quad e \in A - B - C$$
$$= (G(e) \cup H(e)) \quad \text{if} \quad e \in B \cap C - A$$
$$= (F(e) \cup H(e)) \quad \text{if} \quad e \in A \cap C - B$$
$$= (G(e) \cup F(e)) \quad \text{if} \quad e \in A \cap B - C$$
$$= (F(e) \cup G(e) \cup H(e)) \quad \text{if} \quad e \in A \cap B \cap C$$

Assume that, $(F,A)\tilde{\cup}(G,B)=(K,S)$ where $S=A\cup B$ and $\forall e\in S$,

$$
\begin{aligned}
K(e) &= F(e) && if \quad e\in A-B\\
&= G(e) && if \quad e\in B-A\\
&= F(e)\cup G(e) && if \quad e\in A\cap B
\end{aligned}
$$

$((F,A)\tilde{\cup}(G,B))\tilde{\cup}(H,C)=(K,S)\tilde{\cup}(H,C)$,

we suppose that,

$(K,S)\tilde{\cup}(H,C)=(L,T)$, where, $T=S\cup C=A\cup B\cup C$ and $\forall e\in T$,

$$
\begin{aligned}
L(e) &= G(e) && if \quad e\in B-C-A\\
&= H(e) && if \quad e\in C-B-A\\
&= F(e) && if \quad e\in A-B-C\\
&= (G(e)\cup H(e)) && if \quad e\in B\cap C-A\\
&= (F(e)\cup H(e)) && if \quad e\in A\cap C-B\\
&= (G(e)\cup F(e)) && if \quad e\in A\cap B-C\\
&= (F(e)\cup G(e)\cup H(e)) && if \quad e\in A\cap B\cap C
\end{aligned}
$$

Assume that, $(F,A)\tilde{\cup}(G,B)\tilde{\cup}(H,C)=(Q,F)$

$$
\begin{aligned}
Q(e) &= F(e) && if \quad e\in A-B-C\\
&= G(e) && if \quad e\in B-C-A\\
&= H(e) && if \quad e\in C-A-B\\
&= (F(e)\cup G(e)) && if \quad e\in A\cap B-C\\
&= (G(e)\cup H(e)) && if \quad e\in B\cap C-A\\
&= (H(e)\cup F(e)) && if \quad e\in C\cap A-B\\
&= (F(e)\cup G(e)\cup H(e)) && if \quad e\in A\cap B\cap C
\end{aligned}
$$

Therefore it is clear that $M=T=F$ and $\forall e\in M$, $J(e)=L(e)=Q(e)$, that is J and L and Q are same operators.

Thus,

$(F,A)\tilde{\cup}((G,B)\tilde{\cup}(H,C))=((F,A)\tilde{\cup}(G,B))\tilde{\cup}(H,C)=(F,A)\tilde{\cup}(G,B)\cup(H,C)$

(ii) Suppose that, $(G,B)\tilde{\cap}(H,C)=(I,D)$ where $D=B\cap C$ and $\forall e\in D$,

$$
\begin{aligned}
I(e) &= G(e) && if \quad e\in B-C\\
&= H(e) && if \quad e\in C-B\\
&= G(e)\cap H(e) && if \quad e\in B\cap C
\end{aligned}
$$

Since $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C))=(F,A)\tilde{\cap}(I,D)$, we suppose that,

$(F,A)\tilde{\cap}(I,D)=(J,M)$, where, $M=A\cap D=A\cap B\cap C$ and $\forall e\in M$,

$$
\begin{aligned}
J(e) &= G(e) && if \quad e\in B-C-A\\
&= H(e) && if \quad e\in C-B-A\\
&= F(e) && if \quad e\in A-B-C\\
&= (G(e)\cap H(e)) && if \quad e\in B\cap C-A\\
&= (F(e)\cap H(e)) && if \quad e\in A\cap C-B\\
&= (G(e)\cap F(e)) && if \quad e\in A\cap B-C\\
&= (F(e)\cap G(e)\cap H(e)) && if \quad e\in A\cap B\cap C
\end{aligned}
$$

Assume that, $(F,A)\tilde{\cap}(G,B)=(K,S)$ where $S=A\cap B$ and $\forall e\in S$,

$$K(e)=F(e) \quad if \quad e\in A-B$$
$$=G(e) \quad if \quad e\in B-A$$
$$=F(e)\cap G(e) \quad if \quad e\in A\cap B$$

$((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)=(K,S)\tilde{\cap}(H,C)$,

we suppose that,

$(K,S)\tilde{\cap}(H,C)=(L,T)$, where, $T=S\cap C=A\cap B\cap C$ and $\forall e\in T$,

$$L(e)=G(e) \quad if \quad e\in B-C-A$$
$$=H(e) \quad if \quad e\in C-B-A$$
$$=F(e) \quad if \quad e\in A-B-C$$
$$=(G(e)\cap H(e)) \quad if \quad e\in B\cap C-A$$
$$=(F(e)\cap H(e)) \quad if \quad e\in A\cap C-B$$
$$=(G(e)\cap F(e)) \quad if \quad e\in A\cap B-C$$
$$=(F(e)\cap G(e)\cap H(e)) \quad if \quad e\in A\cap B\cap C$$

Assume that, $(F,A)\tilde{\cap}(G,B)\tilde{\cap}(H,C)=(Q,F)$

$$Q(e)=F(e) \quad if \quad e\in A-B-C$$
$$=G(e) \quad if \quad e\in B-C-A$$
$$=H(e) \quad if \quad e\in C-A-B$$
$$=(F(e)\cap G(e)) \quad if \quad e\in A\cap B-C$$
$$=(G(e)\cap H(e)) \quad if \quad e\in B\cap C-A$$
$$=(H(e)\cap F(e)) \quad if \quad e\in C\cap A-B$$
$$=(F(e)\cap G(e)\cap H(e)) \quad if \quad e\in A\cap B\cap C$$

Therefore it is clear that $M=T=F$ and $\forall e\in M$, $J(e)=L(e)=Q(e)$, that is J and L and Q are same operators.

Thus, $(F,A)\tilde{\cap}((G,B)\tilde{\cap}(H,C))=((F,A)\tilde{\cap}(G,B))\tilde{\cap}(H,C)=(F,A)\tilde{\cap}(G,B)\tilde{\cap}(H,C)$

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