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FUZZY SOFT GAMMA REGULAR SEMIGROUPS

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Abstract:

In this paper, we have discussed about the fuzzy soft Γ-regular and Γ-intra regular semigroups and their properties.

Index Terms: Soft semigroups, soft Γ-ideals, Γ-regular semigroups, Γ-intra regular semigroups, soft Γ-regular semigroups & fuzzy soft Γ- intra regular semigroups.

1. Introduction:

The concept of fuzzy set was introduced by Zadeh [15] in 1965. Sen and Saha [13] defined the Γ-semigroup in 1986. Soft set theory proposed by Molotsov [8] in 1999. Soft set theory has been applied in many fields. Maji et al [7] worked on soft set theory and fuzzy soft set theory. Ali et al [1] introduced new operations on soft sets. Chinram and Jirojkul [4] defined the bi-ideals in Γ-semigroups in 2007. Prince et al [12] presented the fuzzy Γ- bi- ideals in Γ-semigroups in 2009. Kehayopula et al [6] initiated the bi-ideals and quasi ideals of semigroups and ordered semigroups. Dheena et al [5] studied the characterization of regular Γ-semigroups through fuzzy ideals in 2007. Chinnadurai et al [3] worked on interval valued fuzzy soft semigroups, and he studied Γ-regular semigroups in fuzzy ideals in algebraic structures. Muhammad Ifran Ali [10] studied soft ideals over semigroups. Sujit Kumar Sardar [14] discussed the properties of fuzzy ideals in Γ-semigroups. Muhammad Akaram et al [9] discussed Fuzzy soft Γ semigroups. Munazza Naz et al [11] worked fuzzy soft quasi- ideals on fuzzy soft semigroups Ali et al [2] studied soft ideals and generalized fuzzy ideals in semigroups in 2009.

2. Preliminaries:

Definition 2.1 [13]: Let $S=\{a,b,c,...\}$ and $\Gamma=\{\alpha,\beta,\gamma,...\}$ be two non-empty sets. Then S is called a $\Gamma-$semigroup if it satisfies the conditions

(i) $a\alpha b\in S$,

(ii) $(a\beta b)\gamma c=a\beta(b\gamma c)$ $\quad\forall a,b,c\in S$ and $\alpha,\beta,\gamma\in\Gamma$.

Definition 2.2 [5]: A $\Gamma-$semigroup S is called a regular if for each element $a\in S$, there exists $x\in S$ and $\alpha,\beta\in\Gamma$ such that $a=a\alpha x\beta a$.

Definition 2.3 [8]: Let U be the universel set, E be the set of parameters, $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F,A) is called a soft set over U, where F is mapping given by $F:A\to P(U)$.

Definition 2.4 [7]: Let (F,A) and (G,B) be two soft sets over a common universe U then (F,A) AND (G,B) denoted by $(F,A)\wedge(G,B)$ is defined as

$(F,A)\wedge(G,B)=(H,A\times B)$ where $H(\alpha,\beta)=F(\alpha)\cap G(\beta)$, $\forall(\alpha,\beta)\in A\times B$

Definition 2.5 [7]: Let (F,A) and (G,B) be two soft sets over a common universe U then (F,A) OR (G,B) denoted by $(F,A)\vee(G,B)$ is defined as $(F,A)\vee(G,B)=(H,A\times B)$ where $H(\alpha,\beta)=F(\alpha)\cup G(\beta)$ $\forall(\alpha,\beta)\in A\times B.$

Definition 2.6 [1]: The extended union of two fuzzy soft sets (F,A) and (G,B) over a common universe U is fuzzy soft set denoted by $(F,A)\cup_{\in}(G,B)$ defined as

$(F,A)\cup_{\in}(G,B)=(H,C)$ where $C=A\cup B$, $\forall c\in C.$

$$H(c)=\begin{cases}F(c) & if\ c\in A-B\\ G(c) & if\ c\in B-A\\ F(c)\cup G(c) & if\ c\in A\cap B.\end{cases}$$

Definition 2.7 [1]: The extended intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is fuzzy soft set denoted by $(F,A)\cap_{\epsilon}(G,B)$ defined as $(F,A)\cap_{\epsilon}(G,B)=(H,C)$ where $C=A\cup B,\ \forall c\in C.$

$$H(c)=\begin{cases}F(c) & if\ c\in A-B\\ G(c) & if\ c\in B-A\\ F(c)\cap G(c) & if\ c\in A\cap B.\end{cases}$$

Definition 2.8 [1]: Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\phi$. The restricted intersection of (F,A) and (G,B) is denoted by $(F,A)\cap_R(G,B)$ and defined as $(F,A)\cap_R(G,B)=(H,C)$ where $C=A\cap B,\ \forall c\in C,$ where $H(c)=F(c)\cap G(c).$

Definition 2.9 [2]: The restricted product (H,C) of two fuzzy soft sets (F,A) and (G,B) over a semigroup S is defined as $(H,C)=(F,A)\tilde{\circ}(G,B)$ where $C=A\cap B$ by $H(c)=F(c)\tilde{\circ}G(c),\forall c\in C.$

Definition 2.10 [3]: A soft set (F,A) is called a soft semigroup over S if $(F,A)\tilde{\circ}(F,A)\subseteq(F,A)$. Clearly a soft set (F,A) over a semigroup S is a soft semigroup if and only if $\phi\neq F(a)$ is a subsemigroup of S, $\forall a\in A.$

Definition 2.11 [10]: A soft semigroup (F,A) over a semigroup S is called a soft regular semigroup if for each $\alpha\in A, F(\alpha)$ is regular.

Definition 2.12 [15]: Let X be non-empty set. A fuzzy subset μ of X is a function from X into the closed unit interval [0, 1]. The set of all fuzzy subsets of X is called a fuzzy power set of X and is denoted by $FP(X)$.

Definition 2.13 [9]: A fuzzy soft set (μ,A) of a $\Gamma-$semigroup S, then (μ,A) is called a fuzzy soft $\Gamma-$subsemigroup of S if $\mu_a(x\gamma y)\geq\min\{\mu_a(x),\mu_a(y)\},\forall a\in A, x,y\in S,\gamma\in\Gamma.$

Definition 2.14 [9]: A fuzzy soft set (μ,A) of a $\Gamma-$semigroup S is called a fuzzy soft $\Gamma-$left(right) ideal of S if $\mu_a(x\gamma y)\geq\mu_a(y)$, $(\mu_a(x\gamma y)\geq\mu_a(x)),\forall a\in A,\ x,y\in S$ and $\gamma\in\Gamma$

Definition 2.15 [9]: A fuzzy soft $\Gamma-$subsemigroup (μ,A) of a $\Gamma-$semigroup S is called a fuzzy soft $\Gamma-$ideal of S if $\mu_a(x\alpha z\beta y)\geq\max\{\mu_a(x),\mu_a(y)\},\forall a\in A,\ x,y,z\in S,\alpha,\beta\in\Gamma.$

Definition 2.16 [9]: A fuzzy soft $\Gamma-$subsemigroup (μ,A) of a $\Gamma-$semigroup S is called a fuzzy soft $\Gamma-$bi-ideal of S if $\mu_a(x\alpha z\beta y)\geq\min\{\mu_a(x),\mu_a(y)\},\forall a\in A,\ x,y,z\in S,\alpha,\beta\in\Gamma.$

Definition 2.17 [9]: A fuzzy soft $\Gamma-$subsemigroup (μ,A) of a $\Gamma-$semigroup S is called a fuzzy soft $\Gamma-$interior ideal of S if $\mu_a(x\alpha z\beta y)\geq\mu_a(z),\forall a\in A,\ x,y,z\in S$ and $\alpha,\beta\in\Gamma.$

Definition 2.18 [11]: A fuzzy soft (μ,A) over S is said to be a fuzzy soft quasi ideal of S if $\mu(e)$ is a fuzzy quasi ideal of S $\forall e\in A.$

Definition 2.19 [11]: A fuzzy soft (μ,A) over S is said to be a fuzzy soft generalized bi-ideal of S if $\mu(e)$ is a fuzzy generalized bi- ideal of S $\forall e\in A.$

3. Fuzzy Soft Gamma Regular Semigroups:

In This section S denotes the soft $\Gamma-$regular semigroup.

Definition 3.1: A soft $\Gamma-$semigroup (F,A) over a semigroup S is called a soft $\Gamma-$regular semigroup if for each $\alpha,\beta\in A, F(\alpha,\beta)$ is regular.

Example 3.2: $S=\{a_1,a_2,a_3\}$ and $\Gamma=\{\alpha,\beta\}$ where α,β is defined on S with the following cayley table:

α	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_3	a_3	a_3

β	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

Table-1

Consider $E=\{a_1,a_2,a_3,a_4\}$and $F(a_1)=\{a_1,a_3\}$, $F(a_2)=\{a_2,a_3\}$, $F(a_3)=\{a_1,a_2,a_3\}$ $F(a_4)=\{a_2,a_3,a_4\}$. Hence (F,S) is soft $\Gamma-$regular semigroup.

Theorem 3.3: Let (F,A) and (G,B) be two fuzzy soft $\Gamma-$ideal (bi-ideal, interior ideal) over soft $\Gamma-$regular semigroup S, then $(F,A)\wedge(G,B)$ is fuzzy soft $\Gamma-$ideal (bi-ideal, interior ideal) over soft $\Gamma-$regular semigroup S.

Proof: Let (F,A) and (G,B) be two fuzzy soft $\Gamma-$ideal over soft $\Gamma-$regular semigroup S. Now we defined $(F,A)\wedge(G,B)=(H,C)$ where $C=A\times B$ and $H(a,b)=F(a)\cap G(b)\ \ \forall(a,b)\in C.$

Consider

$$\begin{aligned}\mu_{H(a,b)}(x\alpha y)&=(\mu_{F(a)}\cap\mu_{G(b)})(x\alpha y)\\&=\min\{\mu_{F(a)}(x\alpha y),\mu_{G(b)}(x\alpha y)\}\\&\geq\min\{\max\{\mu_{F(a)}(x),\mu_{F(a)}(y)\},\max\{\mu_{G(b)}(x),\mu_{G(b)}(y)\}\\&=\max\{\min\{\mu_{F(a)}(x),\mu_{G(b)}(x)\},\min\{\mu_{F(a)}(y),\mu_{G(b)}(y)\}\}\\&=\max\{(\mu_{F(a)}\cap\mu_{G(b)})(x),\mu_{F(a)}\cap\mu_{G(b)}(y)\}\\&=\max\{\mu_{H(a,b)}(x),\mu_{H(a,b)}(y)\}\end{aligned}$$

Hence $(F,A)\wedge(G,B)$ is fuzzy soft $\Gamma-$ideal over soft $\Gamma-$regular semigroup S.

Theorem 3.4: Let (F,A) and (G,B) be two fuzzy soft $\Gamma-$ideal (bi-ideal, interior ideal) over soft $\Gamma-$regular semigroup S, then $(F,A)\vee(G,B)$ is fuzzy soft $\Gamma-$ideal (bi-ideal, interior ideal) over soft $\Gamma-$regular semigroup S.

Proof: The proof is straightforward.

Theorem 3.5: Let (F,A) and (G,B) be two fuzzy soft Γ-ideal of S,then $(F,A)\cap_{\in}(G,B)$ is a fuzzy soft $\Gamma-$ideal over S.

Proof: Let (F,A) and (G,B) be two fuzzy soft $\Gamma-$ideal over S, then

$$(F,A)\cap_{\in}(G,B)=(H,C) \text{ where } C=A\cup B,\ \forall c\in C$$

$$H(c)=\begin{cases}F(c) & if\ \ c\in A-B\\ G(c) & if\ \ c\in B-A\\ F(c)\cap G(c) & if\ \ c\in A\cap B.\end{cases}$$

Let $s,t\in S$ and $\alpha\in\Gamma$.

(i)If $c\in A-B$

$\mu_{H(c)}(s\alpha t) = \mu_{F(c)}(s\alpha t)$

$\geq \max\{\mu_{F(c)}(s), \mu_{F(c)}(t)\}$

$= \max\{\mu_{H(c)}(s), \mu_{H(c)}(t)\}$

(ii)If $c \in B - A$

$\mu_{H(c)}(s\alpha t) = \mu_{G(c)}(s\alpha t)$

$\geq \max\{\mu_{G(c)}(s), \mu_{G(c)}(t)\}$

$= \max\{\mu_{H(c)}(s), \mu_{H(c)}(t)\}$

(iii)If $c \in A \cap B$ then $H(c) = \min\{F(c), G(c)\} = \{F(c) \cap G(c)\}$

Now verify that $H(c)(s\alpha t) \geq \max\{H(c)(s), H(c)(t)\}$, $\forall s,t \in S, \alpha \in \Gamma$ and $c \in C$. Thus $\mu_{H(c)}(s\alpha t) \geq \max\{\mu_{H(c)}(s), \mu_{H(c)}(t)\}$. Hence $(F, A) \cap_{\in} (G, B) = (H, C)$ is a fuzzy soft $\Gamma -$ ideal over S.

Theorem 3.6: Let (F, A) and (G, B) be two fuzzy soft $\Gamma -$ideal over S, $(F, A) \cup_{\in} (G, B)$ is a fuzzy soft $\Gamma -$ideal over S.

Proof: The proof is straightforward.

Theorem 3.7: Let (F, A) and (G, B) be two fuzzy soft sets of soft $\Gamma -$regular semigroup S. A_1 and A_2 are two non-empty subsets of S.

$(i) \chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2} = \chi_{F(e)(A_1 \cap A_2)}$

$(ii) \chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2} = \chi_{F(e)(A_1 \Gamma A_2)}$

Proof: Let $p \in S$, if $p \in A_1 \cap A_2$, then $p \in A_1$ and $p \in A_2$. we have

$(\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2})(p) = \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\}$

$= \min\{0,0\}$

$= 0$

$= \chi_{F(e)(A_1 \Gamma A_2)(p)}$

Suppose $p \notin A_1 \cap A_2$, then $p \notin A_1$ and $p \notin A_2$

$(\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2})(p) = \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\}$

$= \min\{0,0\}$

$= 0$

$= \chi_{F(e)(A_1 \Gamma A_2)(p)}$

Let $p \in S$, if $p \in A_1 \Gamma A_2$, then there exists $a_1 \in A_1$, $\gamma \in \Gamma$ and $a_2 \in A_2$,such that $p = a_1 \gamma a_2$

$(\chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2})(p) = \sup_{p = c\gamma d} \min\{\chi_{F(e)A_1}(c), \chi_{F(e)A_2}(d)\}$

$\geq \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\}$

$= \min\{1,1\}$

$= 1$

So $(\chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2})(p) = 1$ since $p \in A_1 \Gamma A_2$, $\chi_{F(e)\,(A_1 \Gamma A_2)}(p) = 1$ .Suppose $p \notin A_1 \Gamma A_2$, then $p \notin a_1 \gamma a_2, a_1 \in A_1$, $\gamma \in \Gamma$ and $a_2 \in A_2$.

$$(\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2})(p) = \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\}$$
$$= \min\{0,0\}$$
$$= 0$$
$$= \chi_{F(e)(A_1\Gamma A_2)(p)}$$

Hence $\chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2} = \chi_{F(e)\,(A_1\Gamma A_2)}$

Theorem 3.8: Let (F, A) be a soft subset of a semigroup S, (F, A) be a soft $\Gamma-$ subsemigroup of S if and only if $\chi_{F(e)}$ is fuzzy soft $\Gamma-$ subsemigroup of S.

Proof: Let (F, A) be a soft $\Gamma-$ semigroup of S

$$\chi_{F(e)}\lambda(e) = \begin{cases} 1 & if \;\; a \in F(e) \\ 0 & if \;\; a \notin F(e). \end{cases}$$

Let $a, b \in S$, $\gamma \in \Gamma$ if $\chi_{F(e)}\lambda(a\gamma b) \leq \min\{\chi_{F(e)}\lambda(a), \chi_{F(e)}\lambda(b)\}$, then $\chi_{F(e)}\lambda(a) = 1$,

$\chi_{F(e)}\lambda(b) = 1$ and $\chi_{F(e)}\lambda(a\gamma b) = 0$, this implies that $a, b \in F(e)$ since $F(e)$ is $\Gamma-$ subsemigroup of S, $a\gamma b \in F(e)$ and hence $\chi_{F(e)}\lambda(a\gamma b) = 1$ which is a contradiction. Thus $\chi_{F(e)}\lambda(a\gamma b) \leq \min\{\chi_{F(e)}\lambda(a), \chi_{F(e)}\lambda(b)\}, \forall a, b \in F(e)$ and $e \in A$. Conversely assume that $\chi_{F(e)}$ is soft $\Gamma-$ subsemigroup of S. Let $a, b \in F(e)$, then $\chi_{F(e)}\lambda(a) = 1$, $\chi_{F(e)}\lambda(b) = 1$, $\chi_{F(e)}$ fuzzy soft $\Gamma-$ subsemigroup. Now $\min\{\chi_{F(e)}\lambda(a), \chi_{F(e)}\lambda(b)\} = 1 \leq \chi_{F(e)}\lambda(a\gamma b)$, this implies that $\chi_{F(e)}\lambda(a\gamma b) = 1$ and hence $a, b \in F(e)$ $\forall e \in A$. Therefore (F, A) is a soft $\Gamma-$ subsemigroup of S.

Theorem 3.9: Let (F, B) be a soft $\Gamma-$ bi-ideal of S if and only if $\chi_{F(e)B}$ (characteristic function) is a fuzzy soft $\Gamma-$bi-ideal of S.

Proof: Assume that (F, B) be a soft $\Gamma-$bi-ideal of S, by theorem (3.7) wehave $\chi_{F(e)B} \tilde{\circ} S \tilde{\circ} \chi_{F(e)B} = \chi_{F(e)\,(B\Gamma S\Gamma B)} = \chi_{F(e)B,}$ hence $\chi_{F(e)B}$ is a fuzzy soft $\Gamma-$bi-ideal of S. Conversely assume that $\chi_{F(e)B}$ is a fuzzy soft $\Gamma-$bi-ideal of S, $B \subseteq S$, by theorem (3.8), it is clear that B is a soft subsemigroup of S.

Let $p \in S$ such that $p \in B\Gamma S\Gamma B$, then $\chi_{F(e)B}$ is a fuzzy soft $\Gamma-$bi-ideal, we have $\chi_{F(e)B}(p) = (\chi_{F(e)B} \tilde{\circ} S \tilde{\circ} \chi_{F(e)B}) = \chi_{F(e)\,(B\Gamma S\Gamma B)}(p) = 1$ which implies that $p \in B$ and hence $B\Gamma S\Gamma B \subseteq B$. Hence (F, B) is a soft $\Gamma-$bi-ideal of S.

Theorem 3.10: Let (F, A) be a soft $\Gamma-$ideal of S if and only if $\chi_{F(e)A}$ (characteristic function) is a fuzzy soft $\Gamma-$ideal of S.

Proof: The proof is straightforward.

The following theorem is relation between soft set and fuzzy set.

Theorem 3.11: Let (Q, A) be a soft subset of a $\Gamma-$semigroup S, then (Q, A) is a soft Γ-quasi ideal of S if and only if χ_Q (characteristic function) is fuzzy soft Γ -quasi ideal of S.

Proof: Suppose (Q, A) is a soft $\Gamma-$ quasi ideal of S and $\chi_{F(e)}$ be the characteristic function of S, let $x \in S$. If $x \in (Q, A)$ then $((\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q))(x) \leq 1 = f_Q(x)$. If $x \notin (Q, A)$ then $x \notin (Q, A)\Gamma(S, E) \cap (S, E)\Gamma(Q, A) \subseteq (Q, A)$.

Case(i): Let $x \notin (Q, A)\Gamma(S, E)$, $x \in (S, E)\Gamma(Q, A)$. If $x = a\gamma_1 b$ then $a \notin (Q, A)$. Then

$$((\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q))(x) = \min\{\sup_{x=a\gamma_1 b}\{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}, \sup_{x=u\gamma_2 v}\{\min\{\chi_{F(e)}(u), \chi_Q(v)\}\}\}$$
$$= 0 = \chi_Q(x).$$

Therefore $(\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q) \subseteq \chi_Q.$

Case (ii): Let $x \in (Q,A)\Gamma(S,E)$, $x \notin (S,E)\Gamma(Q,A)$. If $x = u\gamma_2 v$, then $v \notin (Q,A)$. Then

$$((\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q))(x) = \min\{\sup_{x=a\gamma_1 b}\{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}, \sup_{x=u\gamma_2 v}\{\min\{\chi_{F(e)}(u), \chi_Q(v)\}\}\}$$
$$= 0 = \chi_Q(x).$$

Therefore $(\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q) \subseteq \chi_Q.$

Case (iii): Let $x \notin (Q,A)\Gamma(S,E)$, $x \notin (S,E)\Gamma(Q,A)$. If $x = a\gamma_1 b$ then $a \notin (Q,A)$, then and if $x = u\gamma_2 v$, then $v \notin (Q,A)$. Then

$$((\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q))(x) = \min\{\sup_{x=a\gamma_1 b}\{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}, \sup_{x=u\gamma_2 v}\{\min\{\chi_{F(e)}(u), \chi_Q(v)\}\}\}$$
$$= 0 = \chi_Q(x).$$

Therefore $(\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q) \subseteq \chi_Q.$ Hence χ_Q is a fuzzy soft $\Gamma-$ quasi ideal of S.

Conversely, suppose that χ_Q is a fuzzy soft $\Gamma-$quasi ideal of S.

Let $x \in (\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q)$ then there exists $s, t \in S$, $y, z \in (Q,A)$ and $\alpha, \beta \in \Gamma$ such that $x = y\alpha s = t\beta z.$

Consider

$$(\chi_Q \tilde{\circ} \chi_{F(e)})(x) = \sup_{x=a\gamma_1 b}\{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}$$
$$\geq \min\{\chi_Q(y), \chi_{F(e)}(s)\}$$
$$= \min\{1,1\}$$
$$= 1$$

Similarly, $(\chi_{F(e)} \tilde{\circ} \chi_Q)(x) = 1.$ Since $(\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q) \subseteq \chi_Q$.

Consider

$$\chi_Q(x) \geq ((\chi_Q \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \chi_Q))(x)$$
$$= \min\{(\chi_Q \tilde{\circ} \chi_{F(e)})(x), (\chi_{F(e)} \tilde{\circ} \chi_Q)(x)\}$$
$$= \min\{1,1\} = 1$$

Thus $x \in (Q,A)$ and hence $(Q,A)\Gamma(S,E) \cap (S,E)\Gamma(Q,A) \subseteq (Q,A)$. Therefore (Q,A) is a soft Γ-quasi ideal of S.

Theorem 3.12: The following conditions are equivalent

(i) Every soft Γ-bi-ideal is a soft Γ-ideal of S. .

(ii) Every fuzzy soft Γ-bi-ideal of S is a fuzzy soft Γ-ideal of S.

Proof: Assume that condition (i) holds, let $\mu_{F(e)}$ be any fuzzy soft Γ-ideal of S. Let $\mu_{F(e)}$ be any fuzzy soft Γ-bi-ideal of S, and $p, q \in S$, since the set $(F,A) \tilde{\circ} (S,E) \tilde{\circ} (F,A)$ is a soft -bi-ideal of S, by the assumption is soft Γ-right ideal of S is soft Γ-regular ,we have $p\Gamma q \in (p\alpha F(\alpha,\beta)\beta\, p)\Gamma q \subseteq p\alpha F(\alpha,\beta)\beta\, p,$ there exists $x \in S$ such that $p\Gamma q = p\alpha x\beta p,$ since $\mu_{F(e)}$ is a fuzzy soft Γ-bi–ideal of S, $\forall\, e \in A.$

Consider

$$\mu_{F(e)}(p\Gamma q) = \mu_{F(e)}(p\alpha x\beta p)$$
$$\geq \min(\mu_{F(e)}(p), \mu_{F(e)}(p))$$
$$= \mu_{F(e)}(p).$$

Hence $\mu_{F(e)}$ is a fuzzy soft Γ-right ideal of S. Similarly $\mu_{F(e)}$ is a fuzzy soft Γ-left ideal of S. Therefore $\mu_{F(e)}$ is a fuzzy soft Γ- ideal of S. Hence $(i) \Rightarrow (ii)$.

Conversely assume that (ii) holds. Let (F, A) be a soft Γ-ideal of S, by theorem (3.9), the characteristic function $\chi_{F(e)B}$ is a fuzzy soft Γ-bi-ideal of S. Hence by assumption $\chi_{F(e)A}$ is a fuzzy soft Γ-ideal of S, thus by theorem (3.10), $\chi_{F(e)A}$ is soft Γ-ideal of S. Hence $(ii) \Rightarrow (i)$

The following examples shows that fuzzy soft Γ-ideal and fuzzy soft Γ-bi-ideal of S.

Examples 3.13: Let $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ in the table.1.

Let $E = \{a_1, a_2, a_3, a_4\}$, $A = \{u_1, u_3\}$ then (F, A) is a fuzzy soft set defined as,

$$\mu_{F(u_1)} = \{(a_1, 0.9), (a_2, 0.3), (a_3, 0.7), (a_4, 0.3)\}$$
$$\mu_{F(u_3)} = \{(a_1, 0.8), (a_2, 0.1), (a_3, 0.5), (a_4, 0.1)\}$$

Hence (F, A) is a fuzzy soft Γ-bi-ideal and fuzzy soft Γ-ideal over S.

Theorem 3.14: The following conditions are equivalent.

(i) (F, A) is a fuzzy soft Γ-ideal of S.

(ii) (F, A) is a fuzzy soft Γ-interior ideal of S.

Proof: Let $\mu_{F(e)}$ is a fuzzy soft Γ-ideal of S, $\forall e \in A$.

We have $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q\beta r)$, since $\mu_{F(e)}$ is a Γ- left ideal of S.

$\geq \mu_{F(e)}(q)$, since $\mu_{F(e)}$ is a Γ- right ideal of S.

Hence $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q)$ $\forall p, q, r \in S$ and $\alpha, \beta \in \Gamma$.

Conversely assume that $\mu_{F(e)}$ is a fuzzy soft Γ- interior ideal of S. Let $p, q \in S$, since S is a soft Γ- regular semigroups, there exists $x, y \in S$, such that $p = p\alpha x\beta p$ and $q = q\alpha y\beta q$ and $\alpha, \beta \in \Gamma$. Thus we have

$$\mu_{F(e)}(p\Gamma q) = \mu_{F(e)}((p\alpha x\beta p)\Gamma q)$$
$$= \mu_{F(e)}((p\alpha x)\beta p\Gamma q)$$
$$\geq \mu_{F(e)}(p)$$

and

$$\mu_{F(e)}(p\Gamma q) = \mu_{F(e)}(p\Gamma(q\alpha y\beta q))$$
$$= \mu_{F(e)}(p\Gamma q\alpha(y\beta q))$$
$$\geq \mu_{F(e)}(q)$$

Hence proved.

The following example shows that fuzzy soft Γ- ideal and Γ- interior ideal of S.

Examples 3.15: Let $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ in the table.1.

Let $E = \{a_1, a_2, a_3, a_4\}$, $A = \{u_1, u_3\}$ then (F, A) is a fuzzy soft set defined as,

$$\mu_{F(u_1)} = \{(a_1, 0.8), (a_2, 0.2), (a_3, 0.4), (a_4, 0.2)\}$$
$$\mu_{F(u_3)} = \{(a_1, 0.7), (a_2, 0.3), (a_3, 0.6), (a_4, 0.3)\}$$

Hence (F, A) is a fuzzy soft Γ-interior ideal and fuzzy soft Γ-ideal over S.

Theorem 3.16: Every fuzzy soft Γ -quasi ideals are fuzzy soft Γ -bi-ideal of S.

Proof: Let $\mu_{F(e)}$ is a fuzzy soft Γ - quasi ideal of S. It is sufficient to prove that

$\mu_{F(e)}(p\alpha q\beta r) \geq \min\{\mu_{F(e)}(p), \mu_{F(e)}(r)\}\ \forall\ p,q,r \in S, e \in A$ and $\alpha, \beta \in \Gamma$.

since $\mu_{F(e)}$ is a fuzzy soft Γ -quasi ideal of S. Consider

$$\mu_{F(e)}(p\alpha q\beta r) \geq ((\mu_{F(e)} \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \mu_{F(e)}))(p\alpha q\beta r)$$
$$= \min\{(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p\alpha q\beta r), (\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p\alpha q\beta r)\}$$

$$(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p\alpha q\beta r) = \sup_{p\alpha q\beta r = u\Gamma v} \min\{\mu_{F(e)}(u), \chi_{F(e)}(v)\}$$
$$= \min\{\mu_{F(e)}(p), \chi_{F(e)}(q\beta r)\}$$
$$= \mu_{F(e)}(p)$$
$$(\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p\alpha q\beta r) = \sup_{p\alpha q\beta r = u\Gamma v} \min\{\chi_{F(e)}(u), \mu_{F(e)}(v)\}$$
$$= \min\{\chi_{F(e)}(p\alpha q), \mu_{F(e)}(r)\}$$
$$= \mu_{F(e)}(r)$$

Hence $\mu_{F(e)}(p\alpha q\beta r) \geq \min\{\mu_{F(e)}(p), \mu_{F(e)}(r)\}$.

Theorem 3.17: In a soft Γ -regular semigroup S, then fuzzy soft Γ -quasi-ideals and fuzzy soft Γ -bi-ideals are coincide.

Proof: It is remains to prove that every fuzzy soft Γ -bi-ideals are fuzzy soft Γ -quasi-ideals if $\mu_{F(e)}$ be a fuzzy soft Γ -bi-ideal of S, then

$(\mu_{F(e)} \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \mu_{F(e)}) \subseteq \mu_{F(e)}$................$(i)$.

Let $p \in S$, suppose that $(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p) \leq \mu_{F(e)}(p), \forall e \in A.$ Consider

$$(\mu_{F(e)}(p) \geq (\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p)$$
$$\geq \min\{(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p), (\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p)\}$$
$$= ((\mu_{F(e)} \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \mu_{F(e)}))(p)$$

Now suppose that $(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p) \geq \mu_{F(e)}(p)$, then there exists $x, y \in S, p = x\Gamma y$ such that $\min\{(\mu_{F(e)}(x) \tilde{\circ} \chi_{F(e)}(y)\} \geq \mu_{F(e)})(p)$ that is $\mu_{F(e)}(x) \geq \mu_{F(e)})(p)$...................$(ii)$,now we prove that $(\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p) \leq \mu_{F(e)})(p)$,so equation (i) is satisfied. Since

$$(\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p) = \sup_{p=q\Gamma r} \min\{\chi_{F(e)}(q), \mu_{F(e)}(r)\}.$$

Prove that $\min\{(\chi_{F(e)}(q) \tilde{\circ} \mu_{F(e)}(r)\} \leq \mu_{F(e)})(p)\quad \forall\ p,q,r \in S$, since S is soft Γ -regular, there exists $u \in S$ such that $p = p\alpha u\beta p = x\Gamma y\alpha u\beta q\Gamma r$. Then since $\mu_{F(e)}$ is a fuzzy soft Γ -bi-ideal, we have $\mu_{F(e)}(p) = \mu_{F(e)}(x\Gamma y\alpha u\beta q\Gamma r) \geq \min\{\mu_{F(e)}(x), \mu_{F(e)}(r)\}$.

If $\min\{\mu_{F(e)}(x), \mu_{F(e)}(r)\} = \mu_{F(e)}(x)$, then $\mu_{F(e)}(x) \leq \mu_{F(e)}(p)$, which is impossible by equation (ii), thus $\min\{\mu_{F(e)}(x), \mu_{F(e)}(r)\} = \mu_{F(e)}(r)$, then

$\mu_{F(e)}(p) \geq \mu_{F(e)}(r) = \min\{\chi_{F(e)}(q), \mu_{F(e)}(r)\}, \forall e \in A.$

Theorem 3.18: The following conditions are equivalent

(i) (F, A) is Γ -regular.

(ii) $(B,\lambda_1)\cap_R (L,\lambda_2)\subseteq(B,\lambda_1)\tilde{\circ}(L,\lambda_2)$ for every soft Γ-bi-ideal (B,λ_1) and every soft Γ-left ideal (L,λ_2) of S.

Proof: $(i)\Rightarrow(ii)$ By definition $(B,\lambda_1)\tilde{\circ}(L,\lambda_2)=(K_1,\lambda_1\cap\lambda_2)$ where K_1 is a function $K_1:(\lambda_1\cap\lambda_2)\to P(F(\alpha,\beta)), K_1(\alpha,\beta)=B(\alpha,\beta)\Gamma L(\beta)\ \forall\alpha,\beta\in\lambda_1\cap\lambda_2$. Now $(B,\lambda_1)\cap_R (L,\lambda_2)=(K_2,\lambda_1\cap_R\lambda_2)$ where K_2 is a function $\lambda_1\cap_R\lambda_2$ to $P(S)$ such that $K_2(\alpha,\beta)=B(\alpha,\beta)\cap L(\beta)\quad\forall\ \alpha,\beta\in\lambda_1\cap\lambda_2$.
we suppose that $p\in B(\alpha,\beta)\cap L(\beta)$, since $p\in F(\alpha,\beta)$ and $F(\alpha,\beta)$ is regular ,there exists $q\in S$ such that $p=p\alpha q\beta p$ .Now $p\in B(\alpha,\beta), q\beta p\in L(\beta)$, and hence $p=p\alpha q\beta p\in B(\alpha,\beta)\Gamma L(\beta)$. This shows that $(B,\lambda_1)\cap_R (L,\lambda_2)\subseteq(B,\lambda_1)\tilde{\circ}(L,\lambda_2)$.
$(ii)\Rightarrow(i)$ .Suppose that $\lambda_1=\lambda_2=F(\alpha,\beta)$ and B is a function from $F(\alpha,\beta)$ to $P(F(\alpha,\beta))$ defined by $B(p)=p\alpha F^1(\alpha,\beta)\beta p,\ \forall\ p\in F(\alpha,\beta)$ and L is a function from $F(\alpha,\beta)$ to $PF(\alpha,\beta))$ defined by $L(p)=F^1(\alpha,\beta)\beta p,\ \forall\ p\in F(\alpha,\beta)$. Then $(B,F(\alpha,\beta)$ is a soft Γ-bi-ideal and $(L,F(\alpha,\beta)$ is a soft Γ-left ideal over $F(\alpha,\beta)$ by hypothesis $p\in B(p)\cap L(p)=B(p)\Gamma L(p)=p\alpha F^1(\alpha,\beta)\beta p\Gamma F^1(\alpha,\beta)\beta p\subseteq p\alpha F^1(\alpha,\beta)\beta p.$
Therefore (F,A) is soft Γ-regular.

Theorem 3.19: Every fuzzy soft Γ-generalized bi-ideal is a fuzzy soft Γ-bi-ideal of S.

Proof: Let $\mu_{F(e)}$ be any fuzzy soft Γ-generalized bi-ideal of S and let $q\in S$, since S is a soft Γ-regular there exists $x\in S$, such that $q=q\alpha x\beta q$. we have

$$\begin{aligned}\mu_{F(e)}(p\Gamma q)&=\mu_{F(e)}(p\Gamma(q\alpha x\beta p))\\&=\mu_{F(e)}(p\Gamma(q\alpha x)\beta q)\\&\geq\min\{\mu_{F(e)}(p),\mu_{F(e)}(q)\}.\end{aligned}$$

This implies that $\mu_{F(e)}$ is a fuzzy soft Γ-subsemigroup of S and so $\mu_{F(e)}$ is a fuzzy soft Γ-bi-ideal of S.

Theorem 3.20: The following conditions are equivalent

(i) (F,A) is Γ-regular.

(ii) $(B,\lambda_1)\cap_R (L,\lambda_2)\subseteq(B,\lambda_1)\tilde{\circ}(L,\lambda_2)$ for every fuzzy soft Γ-generalized bi-ideal (B,λ_1) and every fuzzy soft Γ-left ideal (L,λ_2) of S.

(iii) $(B,\lambda_1)\cap_R (L,\lambda_2)\subseteq(B,\lambda_1)\tilde{\circ}(L,\lambda_2)$ for every fuzzy soft Γ- bi-ideal (B,λ_1) and every fuzzy soft Γ-left ideal (L,λ_2) of S.

Proof: $(i)\Rightarrow(ii)$ .By definition $(B,\lambda_1)\tilde{\circ}(L,\lambda_2)=(N,\lambda_1\cap\lambda_2)$ where N is a function $N:(\lambda_1\cap\lambda_2)\to P(F(\alpha,\beta)), N(\alpha,\beta)=L(\alpha)\Gamma B(\alpha,\beta)\ \forall\alpha,\beta\in(\lambda_1\cap\lambda_2)$. Let (B,λ_1) be a fuzzy soft generalized Γ -bi-ideal and (L,λ_2) be fuzzy soft Γ- left ideal of S. Since (F,A) is soft Γ- regular, let $a\in F(\alpha,\beta)$ there exists $x\in S$ such that $a=a\alpha x\beta a$. Consider

$$\begin{aligned}(\lambda_1\tilde{\circ}\lambda_2)(a)&=\sup_{a=p\Gamma q}\ \min\{\lambda_1(p),\lambda_2(q)\}\\&\geq\min\{\lambda_1(a),\lambda_2(x\beta a)\}\\&=\min\{\lambda_1(a),\lambda_2(a)\}\\&=(\lambda_1(a)\cap\lambda_2(a))(a)\end{aligned}$$

Which implies that $(B,\lambda_1)\cap_R (L,\lambda_2)\subseteq(B,\lambda_1)\tilde{\circ}(L,\lambda_2)$.
$(ii)\Rightarrow(iii)$. By theorem 3.19.

$(iii) \Rightarrow (i)$. let (B,λ_1) be a fuzzy soft Γ-bi-ideal and (L,λ_2) be fuzzy soft Γ- left ideal of S, let $a \in (B,\lambda_1) \cap_R (L,\lambda_2)$, by theorem (3.10) and theorem (3.9), $\chi_{F(e)L}$ is a fuzzy soft Γ - left ideal and $\chi_{F(e)B}$ is a fuzzy soft Γ- bi-ideal of S, and $(B,\lambda_1)\tilde{\circ}(L,\lambda_2) = (N, \lambda_1 \cap \lambda_2)$ where $N:(\lambda_1 \cap \lambda_2) \to P(F(\alpha,\beta))$ and $N(\alpha,\beta) = L(\alpha)\Gamma B(\alpha,\beta)$ $\forall \alpha,\beta \in (\lambda_1 \cap \lambda_2)$. Then by hypothesis $\chi_{F(e)B} \cap \chi_{F(e)L} \subseteq \chi_{F(e)B} \tilde{\circ} \chi_{F(e)L}$.

we have

$(\chi_{F(e)B} \cap \chi_{F(e)L})(p) = \min\{\chi_{F(e)B}(p), \chi_{F(e)L}(p)\}$

$= \min\{1,1\} = 1.$

$\forall a \in (B,\lambda_1) \cap_R (L,\lambda_2)$, since $\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L}$ is a fuzzy soft subset of S, we have $(\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L})(a) \le 1 \ \forall\ a \in S.$

Consider

$$(\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L})(a) = \sup_{a=p\Gamma q} \min\{\chi_{F(e)B}(p), \chi_{F(e)L}(q)\}$$

$$\ge \min\{\chi_{F(e)B}(a), \chi_{F(e)L}(a)\}$$

$$= \min\{1,1\} = 1.$$

Hence $\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L} = 1$ thus $1 = \sup_{a=p\Gamma q} \min\{\chi_{F(e)B}(p), \chi_{F(e)L}(q)\}$, which implies that $\chi_{F(e)B}(p) = 1$ and $\chi_{F(e)L}(q) = 1$ it follows that $p \in (B,\lambda_1)$, $q \in (L,\lambda_2)$ then $a = p\Gamma q \in (B,\lambda_1)\Gamma(L,\lambda_2)$. Therefore $(B,\lambda_1) \cap_R (L,\lambda_2) \subseteq (B,\lambda_1) \tilde{\circ} (L,\lambda_2)$, by theorem (3.18), hence (F,A) is soft Γ-regular semigroup.

Theorem 3.21: The following conditions are equivalent

(i) (F,A) is left Γ-regular.

(ii) $(I,\lambda_1) \cap_R (B,\lambda_2) \subseteq (I,\lambda_1) \tilde{\circ} (B,\lambda_2)$ for every fuzzy soft Γ-ideal (I,λ_1) and fuzzy soft Γ-bi-ideal (B,λ_2) of S.

Proof: $(i) \Rightarrow (ii)$ By definition $(I,\lambda_1)\tilde{\circ}(B,\lambda_2) = (K, \lambda_1 \cap \lambda_2)$ where K is a function $K:(\lambda_1 \cap \lambda_2) \to P(F(\alpha,\beta)), K(\alpha) = I(\alpha,\beta)\Gamma B(\alpha,\beta)$, $\forall \alpha,\beta \in \lambda_1 \cap \lambda_2$. Let (F,A) be soft $\Gamma-$left regular and $a \in F(\alpha,\beta)$, then there exists $p \in S$ and $\alpha,\beta \in \Gamma$ such that $a = p\alpha a\beta a$.

Let (I,λ_1) be fuzzy soft $\Gamma-$ideal and (B,λ_2) be fuzzy soft $\Gamma-$ bi-ideal of S. Consider

$$(\lambda_1 \tilde{\circ} \lambda_2)(a) = \sup_{a=p\Gamma q} \min\{\lambda_1(p), \lambda_2(q)\}$$

$$\ge \min\{\lambda_1(p\alpha a), \lambda_2(a)\}$$

$$\ge \min\{\lambda_1(a), \lambda_2(a)\}$$

$$= (\lambda_1 \cap \lambda_2)(a)$$

Hence $(I,\lambda_1) \cap_R (B,\lambda_2) \subseteq (I,\lambda_1) \tilde{\circ} (B,\lambda_2)$

$(ii) \Rightarrow (i)$ suppose (I,λ_1) and (B,λ_2) are fuzzy soft $\Gamma-$ ideal and fuzzy soft $\Gamma-$ bi-ideal of S such that $(I,\lambda_1) \cap_R (B,\lambda_2) \subseteq (I,\lambda_1) \tilde{\circ} (B,\lambda_2)$. Let I be any soft $\Gamma-$ ideal and B be any soft $\Gamma-$ bi-ideal of S, $p \in I \cap B$, by theorem (3.10) $\chi_{F(e)I}$ is fuzzy soft $\Gamma-$ ideal of S and by theorem (3.9) $\chi_{F(e)B}$ is fuzzy soft $\Gamma-$bi- ideal of S. Now by theorem (3.7) we have $\chi_{F(e)I \cap B} = \chi_{F(e)I} \cap \chi_{F(e)B}$ and hence

$$1 = (\chi_{F(e)I\cap B})(p)$$
$$= (\chi_{F(e)I} \tilde{\cap} \chi_{F(e)B})(p)$$
$$\leq (\chi_{F(e)I} \tilde{\circ} \chi_{F(e)B})(p)$$
$$= \chi_{F(e)I\Gamma B}(p) \text{ by theorem } (3.7)$$

It follows that $p \in (I,\lambda_1) \tilde{\circ} (B,\lambda_2)$ and hence $(I,\lambda_1) \cap_R (B,\lambda_2) \subseteq (I,\lambda_1) \tilde{\circ} (B,\lambda_2)$. Hence (F,A) is soft left Γ-regular.

4. Fuzzy Soft Gamma Intra Regular Semigroups:

In this section S denotes the soft $\Gamma -$intra regular semigroup.

Definition 4.1: A soft $\Gamma -$semigroup (F,A) over a semigroup S is called a soft $\Gamma -$intra regular semigroup if for each $\alpha,\beta,\gamma \in A, F(\alpha,\beta,\gamma)$ is intra regular.

Example 4.2: $S = \{a_1,a_2,a_3,a_4\}$, and $\Gamma = \{\alpha,\beta,\gamma\}$ where α,β,γ is defined on S with the following cayley table:

α	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_3	a_3	a_3

β	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_3
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

γ	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_3	a_3	a_3
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

Table-2

Consider $E = \{a_1,a_2,a_3,a_4\}$ and $F(a_1) = \{a_1,a_3\}$, $F(a_2) = \{a_1,a_2,a_3\}$, $F(a_3) = \{a_2,a_3,a_4\}$ $F(a_4) = \{a_1,a_3,a_4\}$ Hence (F,S) is soft $\Gamma -$intra regular semigroup.

Theorem 4.3: Let (F,A) and (G,B) be two fuzzy soft $\Gamma -$ideal (bi-ideal, interior ideal) over soft $\Gamma -$ intra regular semigroup S, then $(F,A) \wedge (G,B)$ is fuzzy soft $\Gamma -$ideal (bi-ideal, interior ideal) over soft $\Gamma -$intra regular semigroup S.

Proof: The proof is Straightforward

Theorem 4.4: Let (F,A) and (G,B) be two fuzzy soft $\Gamma -$ideal (bi-ideal, interior ideal) over soft $\Gamma -$ intra regular semigroup S, then $(F,A) \vee (G,B)$ is fuzzy soft $\Gamma -$ideal (bi-ideal, interior ideal) over soft $\Gamma -$intra regular semigroup S.

Proof: The proof is Straightforward

Theorem 4.5: The following conditions are equivalent.

(i) (F,A) is a fuzzy soft Γ-ideal of S.

(ii) (F,A) is a fuzzy soft Γ-interior ideal of S.

Proof: Let $\mu_{F(e)}$ is a fuzzy soft Γ-ideal of S, $\forall e \in A$.

We have $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q\beta r)$, since $\mu_{F(e)}$ is a Γ - left ideal of S.

$\geq \mu_{F(e)}(q)$, since $\mu_{F(e)}$ is a Γ - right ideal of S.

Hence $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q)$ $\forall p,q,r \in S$ and $\alpha,\beta \in \Gamma$.

Conversely assume that $\mu_{F(e)}$ is a fuzzy soft Γ - interior ideal of S. Let $p,q \in S$, since S is a soft Γ - intra regular semigroups, there exists $x,y,u,v \in S$, such that $p = x\alpha p\beta p\gamma y$ and

$q = u\alpha q\beta q\gamma v$ and $\alpha, \beta, \gamma \in \Gamma$. Thus we have

$$\mu_{F(e)}(p\Gamma q) = \mu_{F(e)}((x\alpha p\beta p\gamma y)\Gamma q)$$
$$= \mu_{F(e)}((x\alpha p)\beta p\gamma(y\Gamma q))$$
$$\geq \mu_{F(e)}(p)$$

and

$$\mu_{F(e)}(p\Gamma q) = \mu_{F(e)}(p\Gamma(u\alpha q\beta q\gamma v))$$
$$= \mu_{F(e)}(p\Gamma u)\alpha q\beta(q\gamma v))$$
$$\geq \mu_{F(e)}(q)$$

Hence proved.

Theorem 4.6: The following conditions are equivalent (i) (F, A) is Γ-intra regular. (ii) $(L,\lambda_1) \cap_R (R,\lambda_2) \subseteq (L,\lambda_1) \tilde{\circ} (R,\lambda_2)$ for every soft Γ-left ideal (L,λ_1) and every soft Γ-right ideal (R,λ_2) of S.

Proof: $(i) \Rightarrow (ii)$ By definition $(L,\lambda_1) \tilde{\circ} (R,\lambda_2) = (K_1, \lambda_1 \cap \lambda_2)$ where K_1 is a function $K_1 : (\lambda_1 \cap \lambda_2) \to P(F(\alpha,\beta)), K_1(\Gamma) = L(\alpha)\Gamma R(\gamma)\ \ \forall\ \Gamma \in \lambda_1 \cap \lambda_2$. we have $(L,\lambda_1) \cap_R (R,\lambda_2) = (K_2, \lambda_1 \cap_R \lambda_2)$ where K_2 is a function $\lambda_1 \cap_R \lambda_2$ to $P(S)$ such that $K_2(\alpha,\beta) = L(\alpha)\Gamma R(\gamma)\ \ \forall\ \Gamma \in \lambda_1 \cap \lambda_2$.

we suppose that $a \in L(\alpha) \cap R(\gamma)$, since $a \in F(\Gamma)$ and $F(\Gamma)$ is intra regular ,there exists $p, q \in S$ such that $a = p\alpha a\beta a\gamma q$ .Now $p\alpha a \in L(\alpha), a\gamma q \in R(\gamma)$, and hence $a = p\alpha a\beta a\gamma q \in L(\alpha)\Gamma R(\gamma)$ This shows that $(L,\lambda_1) \cap_R (R,\lambda_2) \subseteq (L,\lambda_1) \tilde{\circ} (R,\lambda_2)$

$(ii) \Rightarrow (i)$ .Suppose that $\lambda_1 = \lambda_2 = F(\Gamma)$ and N is a function from $F(\Gamma)$ *to* $P(F(\Gamma))$ defined by $L(a) = F^1(\Gamma)\alpha a,\ \forall a \in F(\Gamma)$ and R is a function from $F(\Gamma)$ *to* $PF(\Gamma))$ defined by $R(a) = a\gamma F^1(\Gamma),\ \forall a \in F(\Gamma)$. Then $(L, F(\Gamma))$ is a soft Γ-left ideal and $(R, F(\Gamma))$ is a soft Γ-right ideal over $F(\Gamma)$ by hypothesis

$a \in L(a) \cap R(a) = L(a)\Gamma R(a) = F^1(\Gamma)\alpha a\beta a\gamma F^1(\Gamma)$.

Therefore (F, A) is soft Γ-intra regular.

Theorem 4.7: The following conditions are equivalent

(i) (F, A) is left $\Gamma-$intra regular.

(ii) $(L,\lambda_1) \cap_R (R,\lambda_2) \subseteq (L,\lambda_1) \tilde{\circ} (R,\lambda_2)$ for every fuzzy soft $\Gamma-$left ideal (L,λ_1) and fuzzy soft $\Gamma-$right ideal (R,λ_2) of S.

Proof: $(i) \Rightarrow (ii)$ By definition $(L,\lambda_1) \tilde{\circ} (R,\lambda_2) = (M_1, \lambda_1 \cap \lambda_2)$ where M_1 is a function $M_1 : (\lambda_1 \cap \lambda_2) \to P(F(\Gamma)), M_1(\Gamma) = L(\alpha)\Gamma R(\gamma),\ \forall\ \Gamma \in \lambda_1 \cap \lambda_2$.

$(L,\lambda_1) \cap_R (R,\lambda_2) = (M_2, \lambda_1 \cap_R \lambda_2)$ where M_2 is a function $(\lambda_1 \cap \lambda_2) \to P(S)$ such that $M_2(\Gamma) = L(\alpha) \cap_R R(\gamma)\ \ \forall\ \Gamma \in (\lambda_1 \cap \lambda_2)$ Let (F, A) be soft $\Gamma-$intra regular and $a \in F(\Gamma)$, then there exists $p, q \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a = p\alpha\, a\beta\, a\gamma q$. Consider

$$(\lambda_1 \tilde{\circ} \lambda_2)(a) = \sup_{a = p\Gamma q} \min\{\lambda_1(p), \lambda_2(q)\}$$
$$\geq \min\{\lambda_1(p\alpha a), \lambda_2(a\gamma q)\}$$
$$\geq \min\{\lambda_1(a), \lambda_2(a)\}$$
$$= (\lambda_1 \cap \lambda_2)(a)$$

Hence $(L,\lambda_1) \cap_R (R,\lambda_2) \subseteq (L,\lambda_1) \tilde{\circ} (R,\lambda_2)$

$(ii) \Rightarrow (i)$ suppose (L,λ_1) and (R,λ_2) are fuzzy soft $\Gamma-$ left ideal and fuzzy soft $\Gamma-$

right ideal of S such that $(L,\lambda_1)\cap_R (R,\lambda_2)\subseteq (L,\lambda_1)\tilde{\circ}(R,\lambda_2)$, and $a\in (L,\lambda_1)\cap_R (R,\lambda_2)$, by theorem (3.10) $\chi_{F(e)L}$ is fuzzy soft $\Gamma-$ left ideal of S and $\chi_{F(e)R}$ is fuzzy soft $\Gamma-$right ideal of S. Now by theorem (3.7), we have $\chi_{F(e)L\cap R}=\chi_{F(e)L}\tilde{\cap}\chi_{F(e)R}$ and hence

$$1=(\chi_{F(e)L\cap R})(a)$$
$$=(\chi_{F(e)L})(a)\tilde{\cap}(\chi_{F(e)R})(a)$$
$$=(\chi_{F(e)L}\tilde{\circ}\chi_{F(e)R})(a)$$
$$=(\chi_{F(e)L\Gamma R})(a)\ by\ theorem\ (3.7)$$

It follows that $a\in (L,\lambda_1)\tilde{\circ}(R,\lambda_2)$ and hence $(L,\lambda_1)\cap_R (R,\lambda_2)\subseteq (L,\lambda_1)\tilde{\circ}(R,\lambda_2)$ The above theorem hence (F,A) is left $\Gamma-$intra regular.

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