



## TOTAL HARMONIC DISTORTION ANALYSIS OF MATRIX CONVERTER

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### Abstract:

This paper deals with the validation and design analysis of Matrix converter for variable frequency using mathematical equations. The analysis was done using Venturini modulation algorithm. The PI controller is used for Matrix converter to reduce Total Harmonic Distortion (THD) in the output current. The comparative study is done for open loop and closed loop PI compensation in MATLAB-Simulink. Furthermore, the output waveforms are produced with significant reduction in the Total Harmonic Distortion.

**Index Terms:** Matrix converter, PI Controller, Venturini Modulation Algorithm & Total Harmonic Distortion

### Introduction:

Matrix converter is the new development ac to ac converters. It is operated with the bidirectional switch with on/off control. It is a Forced Commutated Cycloconverter (FCC). In Matrix converter the output frequency and amplitude has no limits. There are only nine bidirectional switches used in Matrix converter. The use of storage elements is not required, hence the size and weight of the converter is reduced. Only 9 bidirectional switches are used for the three phase conversion. It provides sinusoidal waveforms for input and output currents. The input power factor is always unity. In Venturini algorithm of Matrix converter the output voltage is restricted to half of its input voltage.

### 1. Topology:

The Matrix converter is a development of the Forced Commutated Cycloconverter (FCC), incorporating PWM voltage control. The switches are controlled in such a way that the average output voltage is a sinusoidal of desired frequency with desired amplitude. It consists of  $3 \times 3$  bidirectional switches [4]. It connects a three phase voltage source with a three phase load, typically motor. Potential practical implementations of the converter have therefore focused on applications where there is control over the design of the motor and where the space and weight are at a premium, such as in integrated motor drives and aerospace and naval applications.

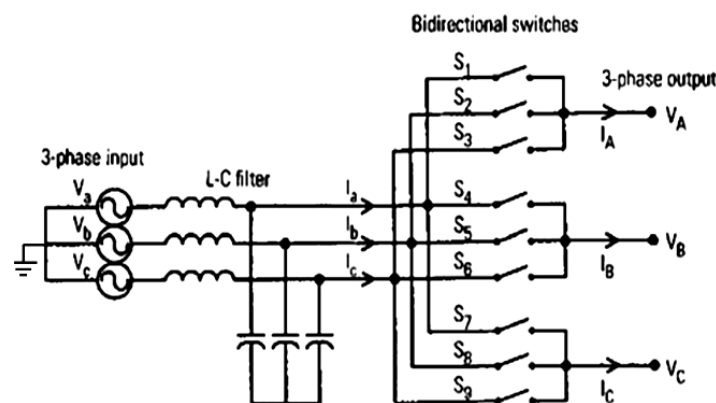


Figure 1.1: Circuit diagram for three phase to three phase Matrix Converter

**A. Bidirectional Switch:** By definition, a bidirectional switch is capable of conducting current and blocking voltages of both polarities, depending on control actual signal. Fig

1.2(a) and 1.2 (b) shows the diagram for the bidirectional switch using diodes and IGBT [3]. When operating with bi-directional switches, two basic rules are to be followed

- ✓ No two input line should be connected to the same output line-to avoid short circuit.
- ✓ At least one of the switches in each phase should be connected to the output-to avoid open circuit

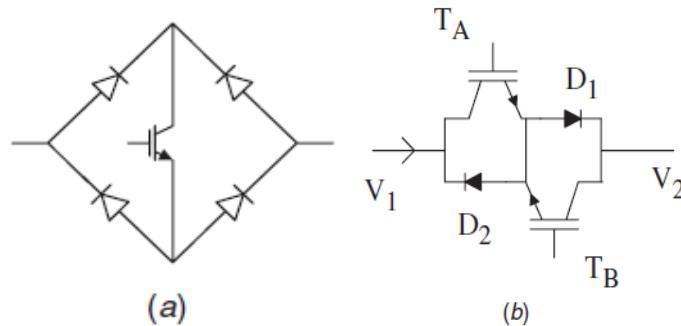


Figure 1.2: (a) Diode bridge bidirectional switch, (b) Common emitter back-to-back bidirectional switch

**B. Switching Algorithm:** With nine bi-directional switches the Matrix converter can theoretically assume 512 different switching states combinations. But all of them cannot be usefully employed, only 27 switching combinations are allowed. These combinations are divided into three groups. Group I consists of six combinations when each output phase is connected to a different input phase. In Group II, there are three sub-groups, each having six combinations with two output phases short-circuited (connected to the same input phase). Group III includes three combinations with all output phases short-circuited [1].

The sequence time is given as

$$T_s = M_{Aa}T_s + M_{Ba}T_s + M_{Ca}T_s \quad (1)$$

Where  $T_s$  is the sequence,  $\alpha = a, b, c$

Where  $f_s = 1/T_s$ , is the switching frequency which should be 20 times higher than the output frequency so as to have low harmonic content in the output voltage. Switches are operated such that the average value of the output voltage is equal to desired voltage during each sampling period  $T_s$  [5].

The switching function of a single switch is given as

$$S_{ij}(t) = \begin{cases} 1, & \text{switchon} \\ 0, & \text{switchoff} \end{cases} \quad (2)$$

The condition can be stated in a more compact form as follows:

$$\sum_{i=A,B,C} S_{ij}(t) = 1; j = \{a, b, c\}, \forall t \quad (3)$$

By applying Kirchoff's voltage law to the switch array, it can be found that

$$\begin{pmatrix} v_{aN}(t) \\ v_{bN}(t) \\ v_{cN}(t) \end{pmatrix} = \begin{pmatrix} S_{Aa}(t) & S_{Ba}(t) & S_{Ca}(t) \\ S_{Ab}(t) & S_{Bb}(t) & S_{Cb}(t) \\ S_{Ac}(t) & S_{Bc}(t) & S_{Cc}(t) \end{pmatrix} \quad (4)$$

$$v_{aN}(t) = S_{Aa}(t)v_a(t) + S_{Ba}(t)v_b(t) + S_{Ca}(t)v_c(t)$$

By applying Kirchoff's current law to the switch array, it can be found that can be generated by modulating the duty cycle of the switches using their respective switching functions. Let  $m_{Kj}(t)$  be the duty cycle of the switch  $S_{Kj}$  defined as  $m_{Kj}(t) = t_{Kj}/T_{seq}$ , which

can have the following values. By considering the bi-directional switches work with high switching frequency, a low frequency output voltages of variable amplitude and frequency

$$0 < mkj < 1k = \{A, B, C\}, j = \{a, b, c\}$$

The lowfrequency transfer matrix is defined by

$$\begin{pmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \end{pmatrix} = \begin{pmatrix} S_{Aa}(t) & S_{Ba}(t) & S_{Ca}(t) \\ S_{Ab}(t) & S_{Bb}(t) & S_{Cb}(t) \\ S_{Ac}(t) & S_{Bc}(t) & S_{Cc}(t) \end{pmatrix} \begin{pmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{pmatrix} \quad (5)$$

$$M(t) = \begin{pmatrix} m_{Aa}(t) & m_{Ba}(t) & m_{Ca}(t) \\ m_{Ab}(t) & m_{Bb}(t) & m_{Cb}(t) \\ m_{Ac}(t) & m_{Bc}(t) & m_{Cc}(t) \end{pmatrix} \quad (6)$$

The low frequency component of the output phase voltage is given by

$$V_o(t) = M(T).V_i(t) \quad (7)$$

The low frequency component of the input current is

$$i_i(t) = M(t)^T \cdot i_o \quad (8)$$

## 2. Modulation Techniques:

A very important solution for the control of Matrix converter comes from the use of pulse width modulation (PWM) techniques previously developed for voltage source inverters. The simplest approach is to use carrier-based PWM technique. The modulation problem normally considered for the Matrix converter can be stated as follows. A set of input voltages and an assumed set of output currents are given

$$V_i(t) = \begin{pmatrix} V_i \cos(\omega t) \\ V_i \cos(\omega t - \frac{2\pi}{3}) \\ V_i \cos(\omega t + \frac{2\pi}{3}) \end{pmatrix} \quad (9)$$

$$i_o(t) = \begin{pmatrix} I_o \cos(\omega_o t + \phi_o) \\ I_o \cos(\omega_o t - \frac{2\pi}{3} + \phi_o) \\ I_o \cos(\omega_o t + \frac{2\pi}{3} + \phi_o) \end{pmatrix} \quad (10)$$

Such that modulation matrix M(t) is

$$V_o(t) = \begin{pmatrix} qV_i \cos(\omega_o t) \\ qV_i \cos(\omega_o t - \frac{2\pi}{3}) \\ qV_i \cos(\omega_o t + \frac{2\pi}{3}) \end{pmatrix} \quad (11)$$

$$i_i(t) = \begin{pmatrix} I_i \cos(\omega_i t + \phi_i) \\ I_i \cos(\omega_i t - \frac{2\pi}{3} + \phi_i) \\ I_i \cos(\omega_i t + \frac{2\pi}{3} + \phi_i) \end{pmatrix} \quad (12)$$

There are two basic solutions given as

$$M1 = \frac{1}{3} \begin{pmatrix} 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t - \frac{2\pi}{3}) & 1+2q\cos(\omega_m t - \frac{4\pi}{3}) \\ 1+2q\cos(\omega_m t - \frac{4\pi}{3}) & 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t - \frac{2\pi}{3}) \\ 1+2q\cos(\omega_m t - \frac{2\pi}{3}) & 1+2q\cos(\omega_m t - \frac{4\pi}{3}) & 1+2q\cos(\omega_m t) \end{pmatrix} \quad (13)$$

With  $\omega_m = (\omega_o - \omega_i)$

$$M2 = \frac{1}{3} \begin{pmatrix} 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t - \frac{2\pi}{3}) & 1+2q\cos(\omega_m t - \frac{4\pi}{3}) \\ 1+2q\cos(\omega_m t - \frac{2\pi}{3}) & 1+2q\cos(\omega_m t - \frac{4\pi}{3}) & 1+2q\cos(\omega_m t) \\ 1+2q\cos(\omega_m t - \frac{4\pi}{3}) & 1+2q\cos(\omega_m t) & 1+2q\cos(\omega_m t - \frac{2\pi}{3}) \end{pmatrix} \quad (14)$$

With  $\omega_m = -(\omega_o + \omega_i)$

The solution in (13) yields  $\Phi_i = \Phi_o$  giving the same displacements at the input and output ports where as the solution in (14) yields  $\Phi_i = -\Phi_o$  giving reverse phase displacement. Combining the two solutions provides the means for input displacement factor control. This basic solution represents a direct transfer function approach and is characterized by the facts that, during each switch sequence time ( $T_{seq}$ ), the average output voltage is equal to the demand (target) voltage. For this to be possible it is clear that the target voltages must fit within the input voltage envelope for any output frequency. This leads to a limitation on the maximum voltage ratio.

#### **A. Venturini Modulation Method:**

The first method attributable to Venturini is defined by equation. However, calculating the switch timings directly from the equation is cumbersome for a practical implementation.

$$M_{kj} = \frac{t_{kj}}{T_{seq}} = \frac{1}{3} \left[ 1 + 2v_k \frac{v_j}{v_{im}^2} \right] \text{ for } k = A, B, C \text{ \& } j = a, b, c \quad (15)$$

This method is of little practical significance because of the 50% voltage ratio limitation. To get the unity input displacement factor the algorithm can be more simply stated in the form of

$$M_{kj} = \frac{1}{3} \left[ 1 + \frac{2v_k v_j}{v_{im}^2} + \frac{4q}{3\sqrt{3}} \sin(\omega_i t + \beta_k) \sin(3\omega_i t) \right] \quad (16)$$

Equation (16) provides a basis for a real time implementation of the optimum amplitude Venturini method which is readily handled processors upto sequence (switching) frequency of tens of KHz. Input displacement factor control can be

introduced by inserting a phase shift between the measured input voltage and the voltages,  $V_k$ , inserted into (16). However, all other methods, displacement factor control is at the expense of maximum voltage ratio.

**3. Mathematical Modeling:**

Mathematical modeling is a unique and valuable resource. Its proper application can yield significant results and deep perception. Modeling of a Matrix converter includes modeling of the power circuit, switching algorithm and load. Advantages of mathematical model over conventional power circuit are:

- ✓ Very less computation time
- ✓ Low memory requirement

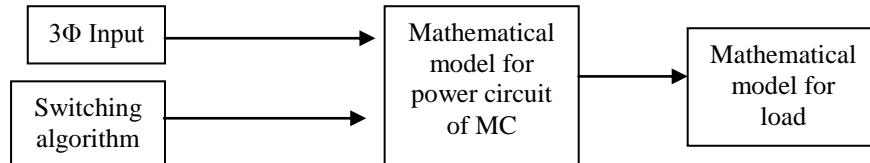


Figure 1.3: Basic block diagram for mathematical model of MC

The block model is simple, flexible and can be accommodated with any type of load. The basic block diagram of mathematical model is shown in Figure 1.3. The mathematical model is validated using a passive RL load. The complete mathematical model of the matrix converter is shown in fig 4.2. for both 0.5 and 0.866 voltage transfer ratio. It comprises of mainly three sections.

- ✓ Control Algorithm
- ✓ Power Circuit
- ✓ Type of Load

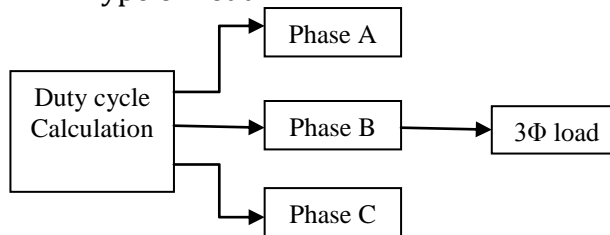


Figure 1.4: Block diagram for mathematical modeling of MC

The required voltage transfer ratio( $q$ ), output frequency( $\omega_o$ ) and switching ( $f_s$ )frequency are the inputs required for calculation of the duty cycle matrix  $M$ . The duty cycle calculations for voltage transfer ratios of 0.5 and 0.866 are realized [2].

Duty cycles calculation for the transfer ratio 0.5 are,

$$\begin{aligned}
 M_{Aa} &= \frac{1}{3}(1 + 2q \cos(\omega_m t)) & M_{Ab} &= \frac{1}{3}(1 + 2q \cos(\omega_m t - \frac{4\pi}{3})) \\
 M_{Ba} &= \frac{1}{3}(1 + 2q \cos(\omega_m t - \frac{2\pi}{3})) & M_{Bb} &= \frac{1}{3}(1 + 2q \cos(\omega_m t)) \\
 M_{Ca} &= \frac{1}{3}(1 + 2q \cos(\omega_m t - \frac{4\pi}{3})) & M_{Cb} &= \frac{1}{3}(1 + 2q \cos(\omega_m t - \frac{2\pi}{3})) \\
 M_{Ac} &= \frac{1}{3}(1 + 2q \cos(\omega_m t - \frac{2\pi}{3})) \\
 M_{Bc} &= \frac{1}{3}(1 + 2q \cos(\omega_m t - \frac{4\pi}{3})) \\
 M_{Cc} &= \frac{1}{3}(1 + 2q \cos(\omega_m t))
 \end{aligned}
 \tag{17}$$

$\omega_m = \omega_o - \omega_i$ , is the modulation frequency.

$\omega_o$ =output frequency

$\omega_i$ =input frequency

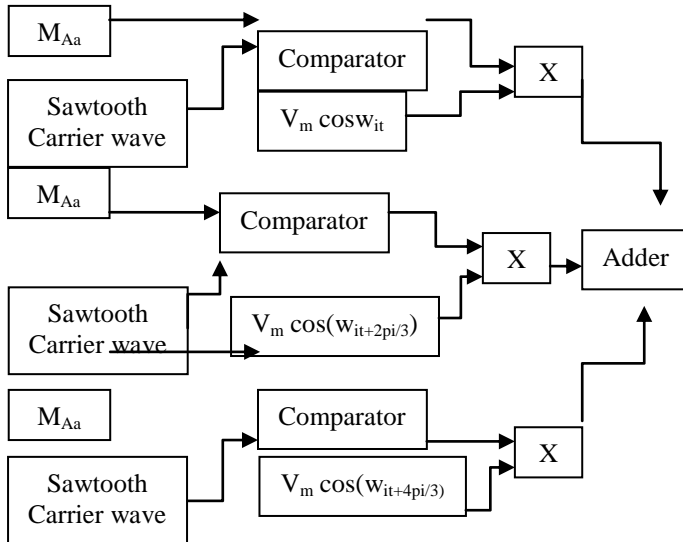


Figure 1.5: Mathematical model of power circuit of “A” phase

The modeling of power circuit is done using the basic equation for the output voltage.

$$\begin{aligned}
 V_a(t) &= M_{Aa}V_A(t) + M_{Ba}V_B(t) + M_{Ca}V_c(t) \\
 V_b(t) &= M_{Ab}V_A(t) + M_{Bb}V_B(t) + M_{Cb}V_c(t) \\
 V_c(t) &= M_{Ac}V_A(t) + M_{Bc}V_B(t) + M_{Cc}V_c(t)
 \end{aligned}
 \tag{18}$$

Realisation of output phase “a” shown in Fig 1.5, the other two phase also can be realized in a similar way. The switching pulse for the bidirectional switches are realized by comparing the duty cycle with a sawtooth waveform having very high frequency.

**B. Closed Loop System:**

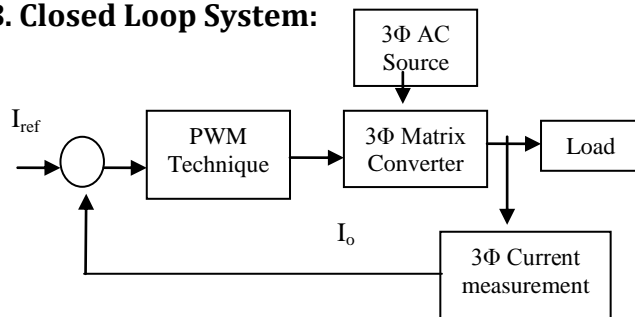


Figure 1.6: Block diagram for MC for closed loop

Modulation algorithms used in the MC have employed fixed switching patterns in sinusoidal input voltages. For certain frequency and amplitude values, duty cycles of the power switches are precalculated and placed into the table. But, since the negative effects of the input voltages reflect the output of the converter under the distorted input voltage conditions, used the fixed switching patterns are not appropriate. Therefore, duty cycles for switching patterns must be calculated instantaneously by measuring the output currents at each switch.

In PI feedback compensation scheme the measured output currents are used to calculate the magnitude of the output current space vector ( $I_o$ ). If the input voltages of the MC are sinusoidal and balanced, the output currents will be sinusoidal, too. Under this condition,  $I_o$  is constant. However, if the input voltages of the MC are non-sinusoidal and unbalanced,  $I_o$  will be not constant due to the output harmonic current.

$$I_o = \sqrt{\frac{2}{3} [i_{oa}^2(t) + i_{ob}^2(t) + i_{oc}^2(t)]} \quad (19)$$

If  $I_o$  is kept constant, the output of the converter is not affected by disturbances in the input voltages. The proposed compensation technique is based on this principle and a PI controller is employed for this purpose [2].

Accordingly, the PI controller system is fed by the instantaneous error of  $I_o(e(k))$  in(2) and produces a variable voltage-gain( $q$ ) according to the disturbance of the input voltage. Duty cycles of the power switches are calculated.

$$e(k) = [I_{ref}(K) - I_{do}(K)] \quad (20)$$

The instantaneous value of the error can be calculated by subtracting the instantaneous current-space-vector  $I_{do}$  obtained by the measured three-phase output current from the reference-current- space-vector  $I_{ref}$ . The output of the PI controller system is the voltage-gain ( $q$ ) and a saturation block has been added to the output of system, due to the magnitude of  $q$  can not exceed 0.866 and can not be negative.

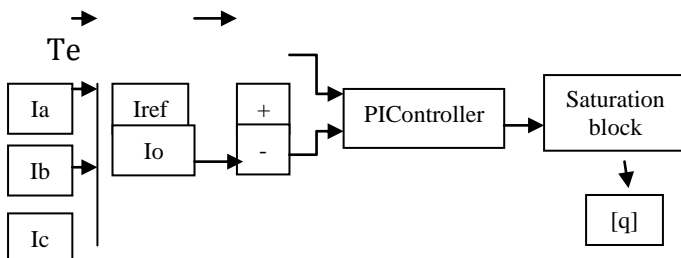


Figure 1.7: Simulink diagram for PI controller based feedback system

**4. Simulation Results:**

The results obtained for the open and closed loop of Matrix Converter is discussed with the waveforms obtained from Matlab Simulink.

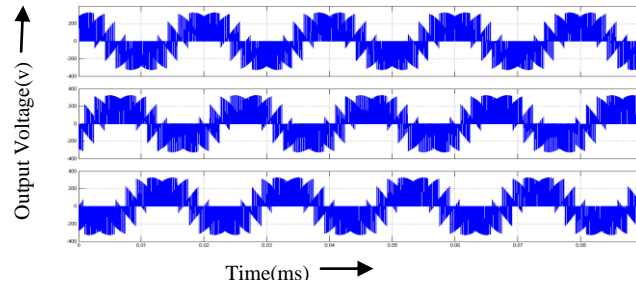


Figure 1.8: Output voltage for closed loop response

The FFT analysis for both open and closed loop response of frequency 50Hz is analysed and compared.

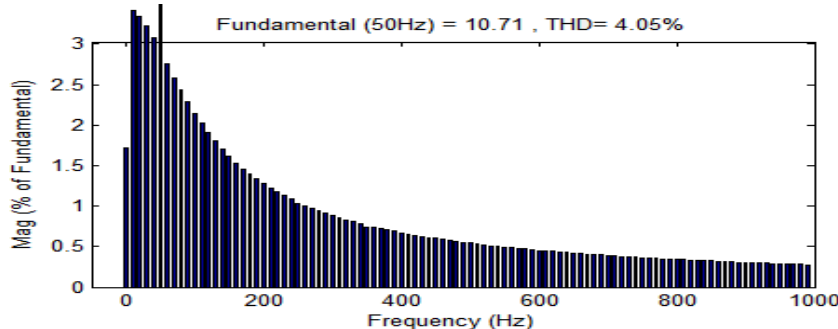


Figure 1.9: FFT analysis of open loop response for frequency 50Hz

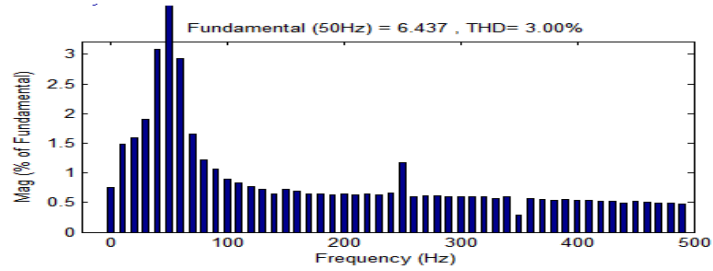


Figure 1.10: FFT analysis of closed loop response for frequency 60Hz  
 The above graph it is clear that THD for open loop response is high than the closed loop response. The comparative analysis was done for both open and closed loop response and for varying frequency.

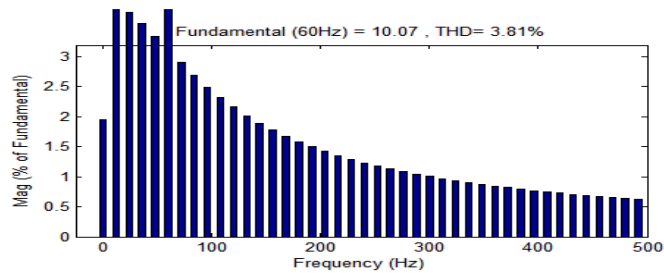


Figure 1.11: FFT analysis for open loop response for frequency 50Hz

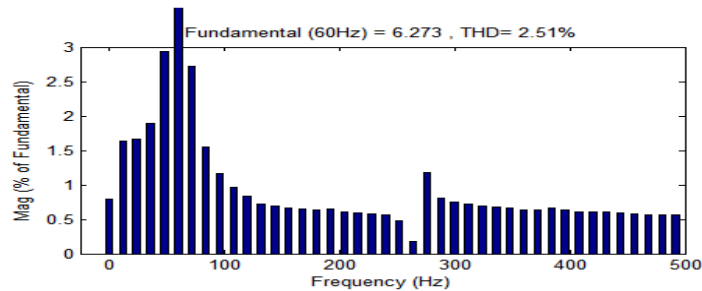


Figure 1.12: FFT analysis for closed loop response for frequency 60Hz  
 From the above analysis it is clear that THD is less for closed loop response than the open loop response

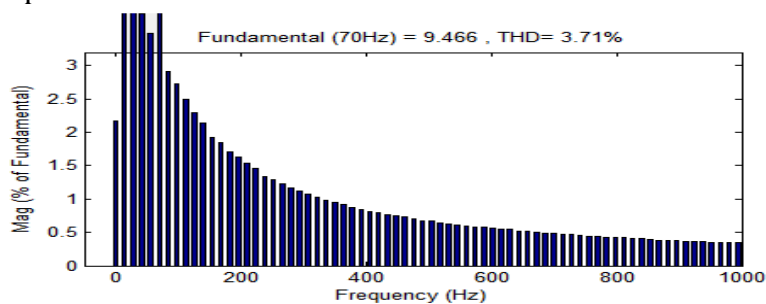


Figure 1.13: FFT analysis for open loop response for frequency 70Hz

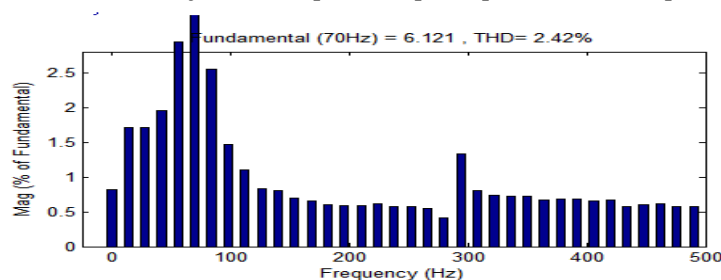


Figure 1.14: FFT analysis for closed loop response for frequency 70Hz



From the FFT analysis for open and closed loop response of different frequencies it is clear that the THD is low in closed loop response than in open loop response.

### **5. Conclusion:**

In this paper the analysis of Matrix converter was done using the mathematical equations. The Venturini modulation technique was used. In Venturini modulation switching pulse can be obtained by comparing the voltage transfer ratio and the saw tooth carrier waveform of switching frequency 10KHz. Simulation has been done for this mathematical model for open and closed loop PI controller response. The above model was implemented using MATLAB/SIMULINK. The voltage transfer ratio and FFT analysis has been compared for open and closed loop response. When compared with open and closed loop response closed loop response is better.

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