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ON PRIME LABELING OF CUBIC GRAPH WITH 8 VERTICES

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Abstract:

In this paper, we show that the cubic graph on 8 vertices admits prime labeling, we also proved that the graphs obtained by merging (or) fusion of two vertices, duplication of an arbitrary vertex and switching of an arbitrary vertex in the cubic graph are prime graphs.

Key Words: Prime Graphs, Fusion, Duplication & Switching

Introduction:

We begin with simple, finite, connected and undirected graph $G = (V, E)$ with p vertices and q edges. For all other standard terminology and notations we refer to J.A. Bondy and U.S.R. Murthy [1]. We give a brief summary of definitions and other information which are useful for the present investigation. A current survey of various graphs labeling problem can be found in [7] (Gallian J, 2009)

Following are the common features of any graph Labeling problem.

- ✓ A set of numbers from which vertex labels are assigned.
- ✓ A rule that assigns value to each edge.
- ✓ A condition that these values must satisfy.

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by A. Tout (1982 P 365-368) [2]. Many researchers have studied prime graph for example in H.C. Fu (1994 P 181-186) [5] have proved that path P_n on n vertices is a prime graph.

T.O. Dertsky (1991 P 359-369) [4] have proved that the cycle C_n on n vertices is a prime graph. S.M. Lee (1998 P 59 -67) [3] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Entringer conjectured that all tress have prime labeling, which is not settled till today. The prime labeling for planner grid is investigated by M. Sundaram (2006 P205-209) [6]. In [8] S. K. Vaidhya and K. K. Kanmani) have proved that the prime labeling for some cycle related graphs. In [9] S. Meena and K. Vaithilingam, Prime Labeling for some Helm related graphs.

Definition 1: If the vertices of the graph are assigned values subject to certain conditions then it is known as (vertex) graph labeling.

Definition 2: Let $G = \big(V(G),\ E(G)\big)$ be a graph with n vertices. A bijection $f: V(G) \to \{1, 2, 3, \ldots\ldots n\}$ is called a Prime labeling if for each edge $e = uv$, $\gcd(f(u), f(v))$. A graph which admits prime labeling is called a prime graph.

Definition 3: An independent set of vertices in a graph G is a set of mutually non-adjacent vertices.

Definition 4: Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by Fusing (identifying) two vertices u and v by a single vertex x in G_1 such that every edge which was incident with either u (or) v in G now incident with x in G_1.

Definition 5: Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words, a vertex v_k' is said to be a Duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k'.

Definition 6: A vertex Switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition 7: A Simple graph G is called a Regular graph if each vertex of G has an equal degree.

Definition 8: A Regular graph G is called a Cubic graph if all the vertices of G are of degree 3.

Illustration:

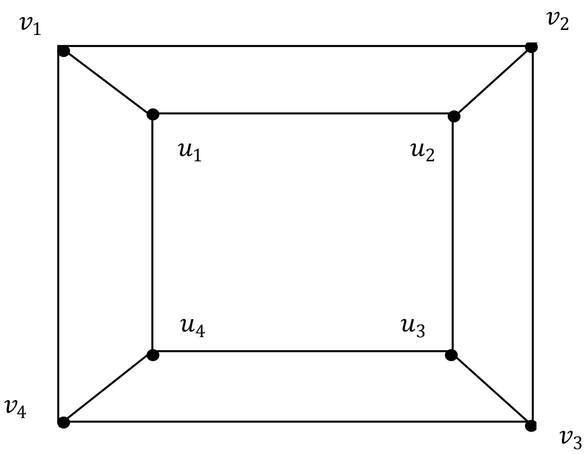

Figure 1: Cubic graph with 8 vertices

Proposition 1:

A Cubic graph with 8 vertices is a prime graph.

Proof:

Let $G = (V, E)$ be a cubic graph with 8 vertices and 12 edges

Let $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$

$E(G) = \{u_i v_i / 1 \leq i \leq 4\} \cup \{u_i u_{i+1} / 1 \leq i \leq 3, u_4 u_1\} \cup \{v_i v_{i+1} / 1 \leq j \leq 3, v_4 v_1\}$

Then $|V(G)| = 8$ and $|E(G)| = 12$

Define a labeling $f: V(G) \rightarrow \{1, 2, 3, \ldots..8\}$

Such that $f(u_i) = i$ for $1 \leq i \leq 4$

(i.e.) $f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_4) = 4$

and $f(v_i) = f(u_i) + 5$ for $1 \leq i \leq 3$

(i.e.) $f(v_1) = f(u_1) + 5 = 1 + 5$

$f(v_1) = 6$

Similarly, $f(v_2) = f(u_2) + 5$

$f(v_2) = 6$

$f(v_3) = f(u_3) + 5$

$f(v_3) = 6$

Finally, $f(v_4) = f(u_4) + 1$

$= 4 + 1$

$f(v_4) = 5$

Clearly, for each edge $e = u_i v_i \in G, \gcd(f(u_i), f(v_i)) = 1$

and the edges $e = u_i u_j, v_i v_j \in G, \gcd\left(f(u_i), f(u_j)\right) = 1$ and $\gcd\left(f(v_i), f(v_j)\right) = 1$

Then G admits a prime labeling

Hence G is a prime graph.

Illustration:

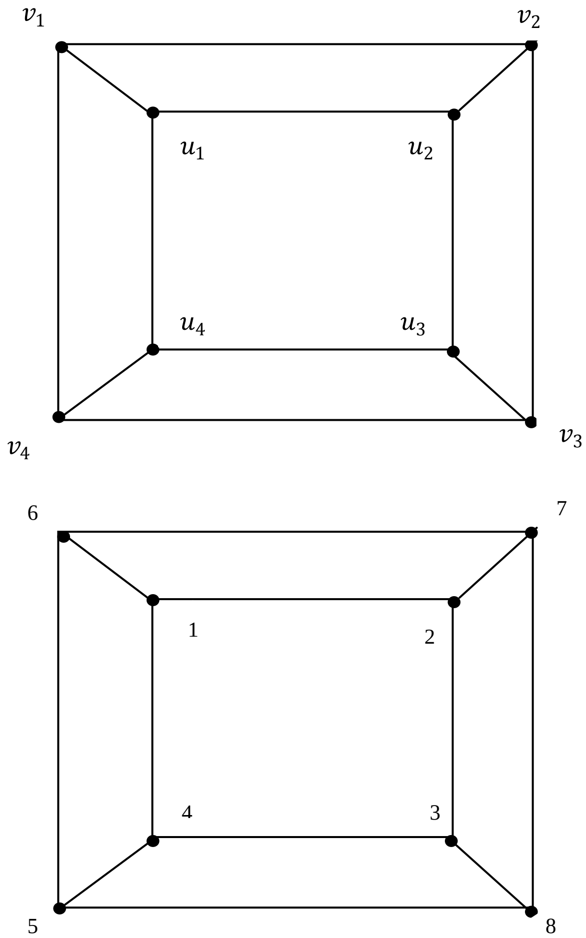

Figure 2: A Cubic graph with 8 vertices is a prime graph.

Remark: A Cubic graph on 4 vertices is not a prime graph.

Proposition 2:

The Fusion of two consecutive vertices in the outer cycle graph on 8 vertices is a prime a graph.

Proof:

Let $G = (V, X)$ be a cubic graph on 8 vertices and G_f be the graph obtained by fusion (or identifying) two vertices v_1 and v_2 (i.e. $v_1 = v_2$) of G.

Then $\left|G_f(V)\right| = 7$

Define a label $f: V(G_f) \to \{1, 2, \dots \dots 7\}$

Such that $f(u_i) = i$ for $1 \le i \le 4$

$f(u_1) = 1$

$f(u_2) = 2$

$f(u_3) = 3$

$f(u_4) = 4$

and $f(v_i) = f(u_i) + 5$ for $1 \le i \le 3$

(i.e.) $f(v_1) = f(u_1) + 5 = 6$

Similarly, $f(v_2 = v_3) = 7$

and $f(v_4) = f(u_4) + 1$

$= 4 + 1$

$f(v_4) = 5$

Clearly, for each edge $e = u_i v_i \in G$, $\gcd(f(u_i), f(v_i)) = 1$

and for the edges $e = u_i v_j$, $v_i v_j \in G$, $\gcd\left(f(u_i), f(u_j)\right) = 1$
and $\gcd\left(f(v_i), f(v_j)\right) = 1$
G_f admits a prime labeling
Hence G_f is a prime graph.

Illustration:

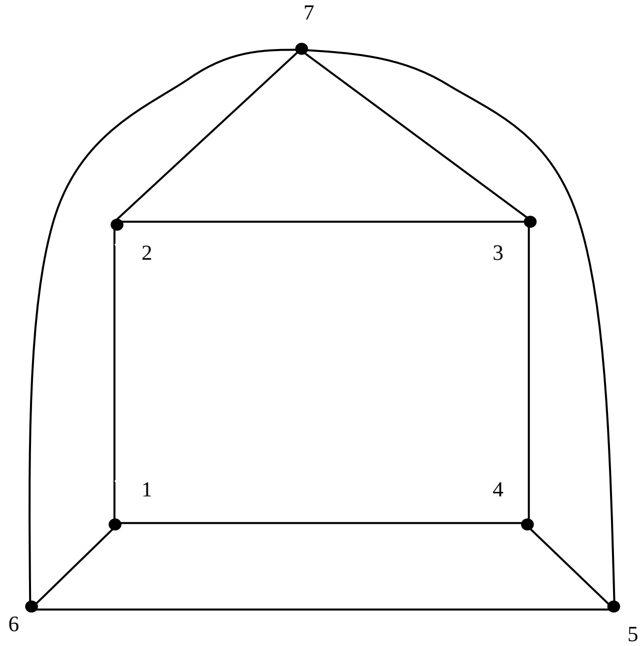

Figure 3: Fusion of two vertices v_2 and v_3 in a Cubic graph is a prime graph.

Proposition 3:

The Duplication of an arbitrary vertex of the Cubic graph on 8 vertices produces a prime graph.

Proof:

Let $G = (V, E)$ be a Cubic graph with 8 vertices and 12 edges
Let $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$
$E(G) = \{u_i v_i / 1 \le i \le 4\} \cup \{u_i u_{i+1}/1 \le i \le 3, u_4 u_1\} \cup \{v_i v_{i+1}/1 \le i \le 3, v_4 v_1\}$

Let G_k be the graph obtained by Duplicating and arbitrary vertex of G. Without loss of generality let this vertex be v_1 and the newly added vertex be v_1'.

Define a label $f: V(G_k) \to \{1, 2, \ldots .9\}$
Such that $f(u_i) = i$ for $1 \le i \le 4$
$f(u_1) = 1$
$f(u_2) = 2$
$f(u_3) = 3$
$f(u_4) = 4$
$f(v_1) = f(u_1) + 5 = 6$ & $f(v_1') = 9$
and $f(v_2) = f(u_2) + 5 = 7$
$f(v_3) = f(u_3) + 5 = 8$
$f(v_4) = f(u_4) + 1 = 5$

Clearly, for the edges $u_i v_i$, $u_i u_j$, $v_i v_j \in G$, $\gcd(f(u_i), f(v_i)) = 1$,
$gcd\left(f(u_i), f(u_j)\right) = 1$ and $gcd\left(f(v_i), f(v_j)\right) = 1$
Then G_k admits a prime labeling
Hence G_k is a prime graph.

Illustration:

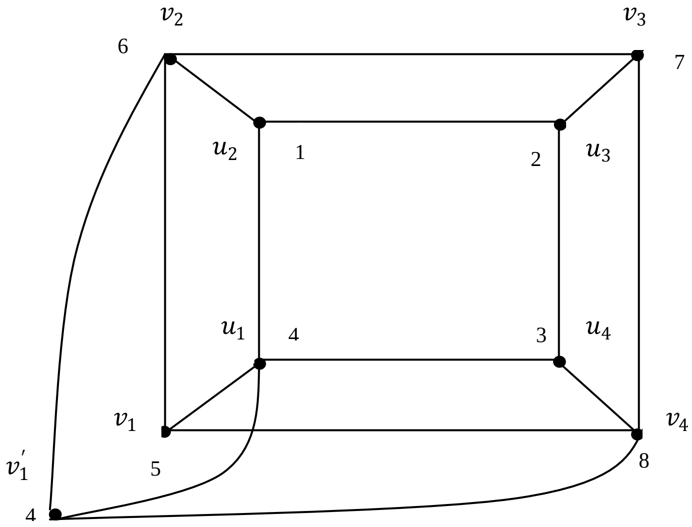

Figure 4: The Duplication of the vertex v_1 in Cubic graph is a prime graph.

Proposition 4:

The Switching of an arbitrary vertex in a cubic graph on 8 vertices is a prime graph.

Proof:

Let $G = (V, E)$ be a Cubic graph with 8 vertices and 12 edges

Let $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$

$E(G) = \{u_i v_i / 1 \leq i \leq 4\} \cup \{u_i u_{i+1}/1 \leq i \leq 3, u_4 u_1\} \cup \{v_i v_{i+1}/1 \leq i \leq 3, v_4 v_1\}$

Let G_s be the graph obtained by switching an arbitrary vertex of G. Without loss of generality let this vertex be v_1 and $|V(G_s)| = 8$ and $|E(G_s)| = 12$

Define a label $f: V(G_s) \to \{1, 2, 3 \ldots \ldots .8\}$ such that

$f(v_i) = i$ for $1 \leq i \leq 4$

$f(v_1) = 1$, where v_1 is a switching vertex

$f(v_2) = 2$

$f(v_3) = 3$

$f(v_4) = 4$

and $f(u_1) = f(v_1) + 5 = 6$

$f(u_2) = f(v_2) + 5 = 7$

$f(u_3) = f(v_3) + 5 = 8$

$f(u_4) = f(v_4) + 1 = 5$

This pattern of labeling admit G_s as prime labeling

Hence G_s is a prime graph.

Illustration:

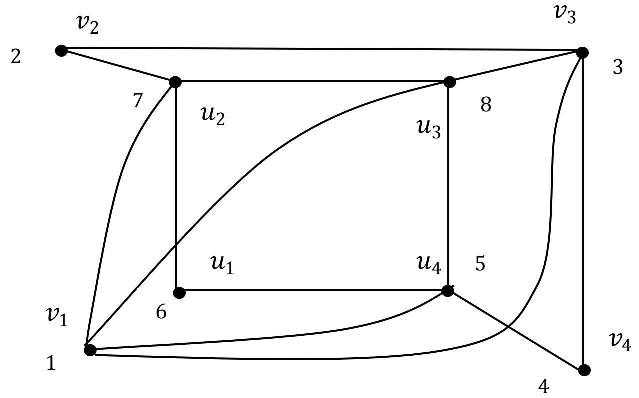

Figure 5: The switching of v_1 in a cubic graph is a prime graph.

Concluding Remarks and Further Scope:

As all graphs are not prime graphs it is very interesting to investigate graphs which admits prime labeling it is possible to investigate similar results for other graph families and in the context of different labeling techniques.

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