



## A MATHEMATICAL MODEL OF THE HAZARD RATE FUNCTION IN FRUCTOSE STIMULATES GLP-1

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### Abstract:

Nutrients often stimulate gut hormone secretion, but the effects of fructose on a number of gut hormones are on glucagon like peptide 1. The fructose intake of humans caused a rise in GLP -1. This paper introduces the hazard rate function of the field time distribution exhibits a range of shapes. The hazard rate function of the distribution is

$$h_T(\tau\alpha) = \frac{\gamma\beta}{[\tau\alpha]}(\tau)^\beta + \frac{\frac{k\beta}{\tau\alpha}}{\left[1 + \frac{\mu\alpha^\beta}{(\tau\alpha)^\beta}\right]}$$

**Key Words:** Glucagon Like Peptide 1, Hazard Rate Function & Frailty

### Introduction:

The Nutrients such as fructose has been suspected to be responsible for the growing rates of obesity and the metabolic syndrome [1] & [8]. The present study was the effect of fructose stimulated GLP-1 secretion in humans.

### Notation:

$\alpha, \beta, \gamma$  - parameters

$h_x(x)$  - hazard Rate function

$\phi(x)$  - three parameters gamma distribution

$F_x(x)$  - Cumulative probability distribution

### Hazard Rate Function of Gamma Frailty Model:

The life time X of the product follows a weibull distribution, which is one of the most commonly used lifetime distributions. The basic idea is to introduce into the hazard rate an random parameter Z that accounts for the heterogeneities. The frailty Z links the distribution of X to that of the field failure time T. The failure time X of a lab testing unit is assumed to follow a weibull distribution with the respective cumulative distribution function is

$$f_x(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right], x > 0 \quad \text{---- (1)}$$

and its probability density function

$$f_x(x) = \frac{\beta}{x}\left(\frac{x}{\alpha}\right)^\beta \exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right] \quad \text{---- (2)}$$

where the parameters  $\alpha > 0, \beta > 0$ .

The hazard rate function of X is given by

$$h_x(x) = \frac{\beta}{x}\left(\frac{x}{\alpha}\right)^\beta \quad \text{---- (3)}$$

The frailty Z is constant for a unit and varies across the product population. The lifetime of a field unit follows the Weibull distribution with a hazard rate function is given by

$$h_T(\tau\alpha, Z) = z \times \frac{\beta}{\alpha} \left(\frac{t}{\alpha}\right)^\beta \quad \text{---- (4)}$$

The distribution of Z depends on the heterogeneities of the field environments as well as the effects of the random environments on the product [4] & [5]. We consider its gamma distribution with threshold parameters in order to demonstrate the fact that the hazard rate function of T exhibits various shapes. The three parameters gamma distribution with a threshold parameter  $\gamma$  has a probability density function given by [15], [16] & [11]

$$\phi(z) = \frac{\mu^k (z - \gamma)^{k-1}}{\Gamma(k)} \exp[-\mu(z - \gamma)], z > \gamma \quad \text{---- (5)}$$

When the frailty follows a distribution in (4) it can be shown by marginalizing Z out of (3) that the cumulative density function and probability density function of T are

$$F_T(\tau\alpha) = 1 - \frac{\mu^k \exp[-\gamma(\tau)^\beta]}{[(\tau)^\beta + \mu]^k} \quad \text{---- (6)}$$

$$f_T(\tau\alpha) = \frac{\frac{\beta}{\tau\alpha} (\tau)^\beta}{\left[\frac{(\tau)^\beta}{\mu} + 1\right]^k} \left\{ \gamma + \frac{k}{[\tau^\beta + \mu]} \right\} \exp[-\gamma(\tau)^\beta] \quad \text{---- (7)}$$

Here if  $\gamma = 0$ , the equation (6) reduces to the Burr -XII distribution

The Burr XII distribution has been used in reliability analysis

$$f_T(\tau\alpha) = \frac{k \frac{\beta}{\tau\alpha} (\tau)^\beta \mu^k}{[\tau^\beta + \mu]^{k+1}} \quad \text{---- (8)}$$

The hazard rate function of z can be obtained by dividing the p.d.f by the survival function, gives

$$h_T(\tau\alpha) = \frac{\gamma\beta}{[\tau\alpha]} (\tau)^\beta + \frac{\frac{k\beta}{\tau\alpha}}{\left[1 + \frac{\mu\alpha^\beta}{(\tau\alpha)^\beta}\right]} \quad \text{---- (9)}$$

Shapes of the hazard rate function of its gamma frailty model we have to taking the first derivative of the hazard rate function of Z [10], [12], [17] & [18]

Depends upon the value of  $\beta > 1$  and  $\beta < 1$  the value of  $h'(t)$  has positive negative positive sign change.

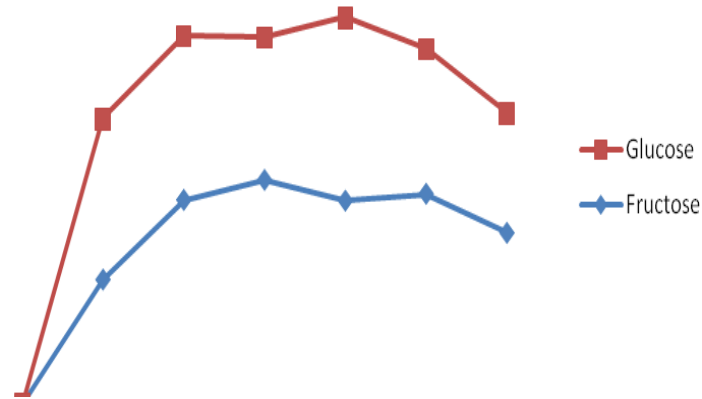
In (9) if  $\gamma = 0$ , gives also the equation of the hazard function

$$h_T(\tau\alpha) = \frac{\frac{k\beta}{\tau\alpha}}{1 + \frac{\mu\alpha^\beta}{(\tau\alpha)^\beta}} \quad \text{---- (10)}$$

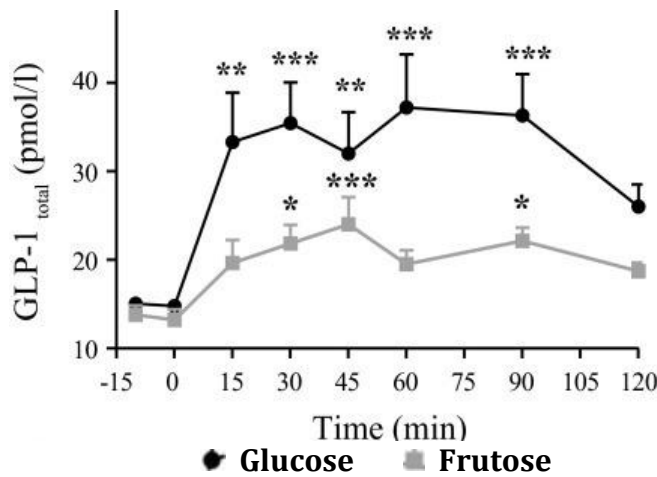
Hazard rates play a fundamental role in survival analysis. Also Hazard rates sometimes increase, sometimes decrease and some times first increase and then decrease [2], [3], [9] & [14].

**Example:**

Nine healthy volunteers participated in the study. All subjects had normal fasting blood glucose levels and none had parents with any type of diabetes. Each participant has a sugar solution containing fructose. [6], [7], [8] & [13]. GLP 1 concentrations increased significantly after oral fructose intake with progressively increasing concentration from base line and also glucose fig(ii)



**Figure (i)**



**Figure (ii)**

**Conclusion:**

The mathematical model also stresses the same cumulative effect of intake of fructose are stimulates GLP-1 which are beautifully filled with Hazard rate function (fig(i)).The results of these analysis indicates that the intake of fructose stimulates GLP - 1 (fig(ii)). The results are exactly related with the Mathematical and medical report.

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