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PRIME CORDIAL LABELING OF SOME GRAPHS

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Abstract:

A Prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f: V(G) \rightarrow \{1,2,\ldots,|V(G)|\}$ such that each edge uv is assigned the label 1 if $gcd\left(f(u),f(v)\right)=1$ and 0 if $gcd\left(f(u),f(v)\right)>1$, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. A graph which admits prime cordial labeling is called prime cordial graph. In this paper an analysis is made on union of graphs are Prime Cordial labeling.

Key Words: Cycle, Union of Graphs, Prime Cordial Labeling & Prime Cordial Graph.

1. Introduction:

A graph labeling is an assignment of labels to edges, vertices or both. Cahit. I [1] introduced the concept of cordial labeling in 1987. J. A Gallian [5] in his dynamic survey, has gathered many types of labeling techniques. Sundaram, Ponraj and Somasundaram [6] introduced concept called prime cordial labeling. In G.V. Ghodasara and J.P. Jena [2] proved prime cordial labeling of cycle with one chord, twin chords and triangle. M. Ibrahim Moussa [4] studied the graph $C_m \cup P_n$ when $m=4,6,8,10$. and then they proved the graph $C_m \cup P_n$ is odd graceful graph if m is even. For the basic notations we refer Harary [3]. In this paper an analysis is made on comb related graphs are Prime Cordial labeling.

2. Definitions:

2.1 Definition:

A Prime cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f: V(G) \rightarrow \{1,2,\ldots,|V(G)|\}$ such that each edge uv is assigned the label 1 if $\gcd\left(f(u),f(v)\right)=1$ and 0 if $\gcd\left(f(u),f(v)\right)>1$, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1. A graph which admits prime cordial labeling is called prime cordial graph.

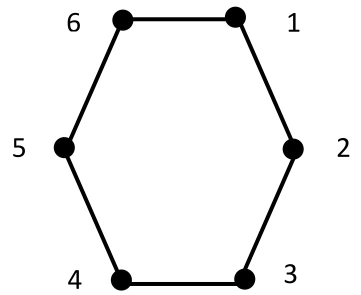

Figure 1: Cycle C_6.

2.2 Definition:

The union of two graphs $G_1=(V_1,E_1)$ and $G_1=(V_2,E_2)$, written $G_1 \cup G_2$, is the graph with vertex set $V(G_1 \cup G_2)=V(G_1)\cup V(G_2)$ and the edge set $E(G_1 \cup G_2)=E(G_1)\cup E(G_2)$.

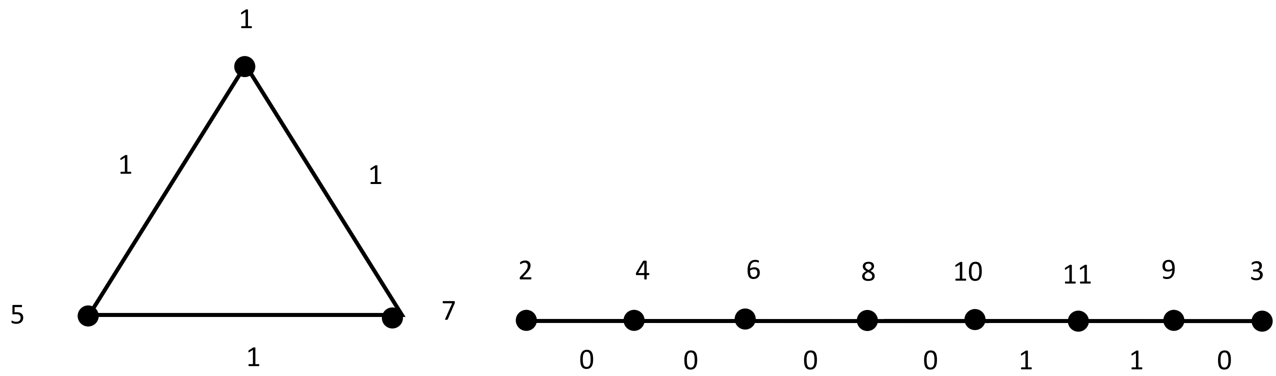

Figure 2: Disconnected graph $C_3 \cup P_8$.

3. Results:

3.1 Theorem:

$C_m \cup P_n$ is prime cordial if $m \leq 5$ is odd and $n \geq 6$.

Proof:

Let $G = (V, E, f)$ be a disconnected graph $C_m \cup P_n$ with order $p = m + n$ and size $q = m + n - 1$. Here $u_1, u_2, .., u_m, v_{1,} v_2, \dots, v_n$ are the vertices where $u_1, u_2, \dots, u_m$ be the vertices of the cycle C_3 and $v_{1,} v_{2,\dots,} v_n$ be the vertices of the path P_n. Then the edges e_i, e_{ij} are defined as $e_i = (u_i, u_{i+1})$ and $e_{ij} = (v_i, v_{i+1})$.

Define the function $f: V(G) \to \{1,2, .., m + n\}$ as follows,

Case (i) $C_3 \cup P_n$

If n is even,

$f(u_1) = 1, f(u_2) = 5, f(u_3) = 7, f(v_n) = 3, f(v_{n-1}) = 9.$

$f(v_i) = 2i, \qquad i = 1.2, \dots, \frac{n+2}{2}.$

$f(v_i) = f(v_{i-1}) + 2, \qquad i = n-2, n-3, \dots, \frac{n+4}{2}.$

If n is odd,

$f(u_1) = 1, f(u_2) = 5, f(u_3) = 7, f(v_n) = 3, f(v_{n-1}) = 9.$

$f(v_i) = 2i, \qquad i = 1,2, \dots, \frac{n+3}{2}.$

$f(v_i) = f(v_{i-1}) + 2 \qquad i = n-2, n-3, \dots, \frac{n+5}{2}.$

Case (ii) $C_5 \cup P_n$

If n is even.

$f(u_1) = 1, f(u_2) = 3, f(u_3) = 9.$

$f(u_{i+2}) = 2i + 1 \quad i = 2.3.$

$f(v_i) = 2i, \qquad i = 1,2, \dots, \frac{n+4}{2}.$

$f(v_n) = 11.$

$f(v_i) = f(v_{i+1}) + 2, \qquad i = n-1, n-2, \dots, \frac{n+6}{2}.$

If n is odd.

$f(u_1) = 1, f(u_2) = 3, f(u_3) = 9.$

$f(u_{i+2}) = 2i + 1, \qquad i = 2.3.$

$f(v_i) = 2i, \qquad i = 1,2, \dots, \frac{n+5}{2}.$

$f(v_n) = 11.$

$$f(v_i) = f(v_{i+1}) + 2,\ \ i = n-1, n-2, \ldots, \frac{n+7}{2}.$$

Then the above function f admits the prime cordial labeling. Hence $C_m \cup P_n$ are prime cordial labeling. The generalized graph of $C_m \cup P_n$ is shown in figure 26.

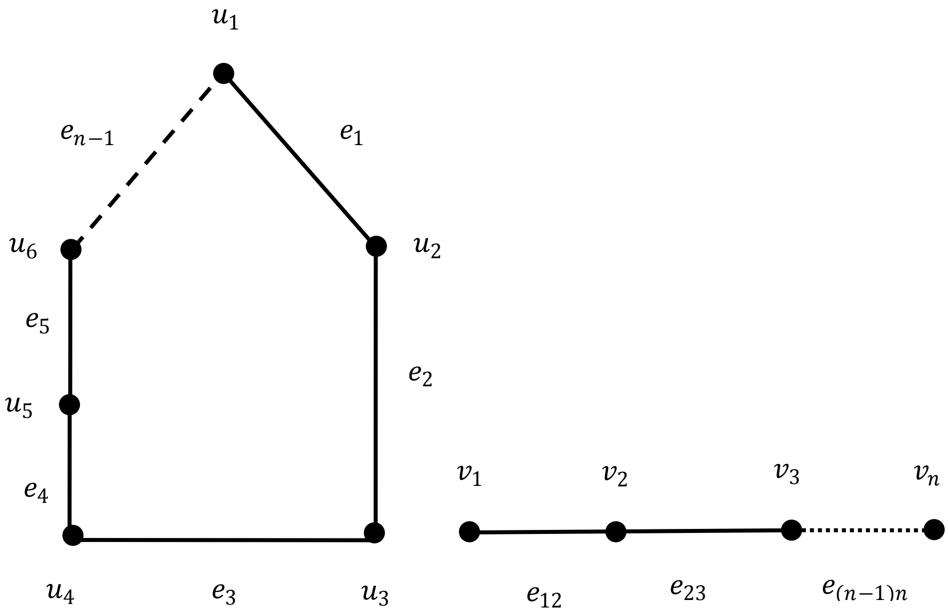

Figure 3: Disconnected graph $C_m \cup P_n$.

3.2 Example:

The graph is given in figure 27 is $C_3 \cup P_6$. The order and size are 9 and 8.

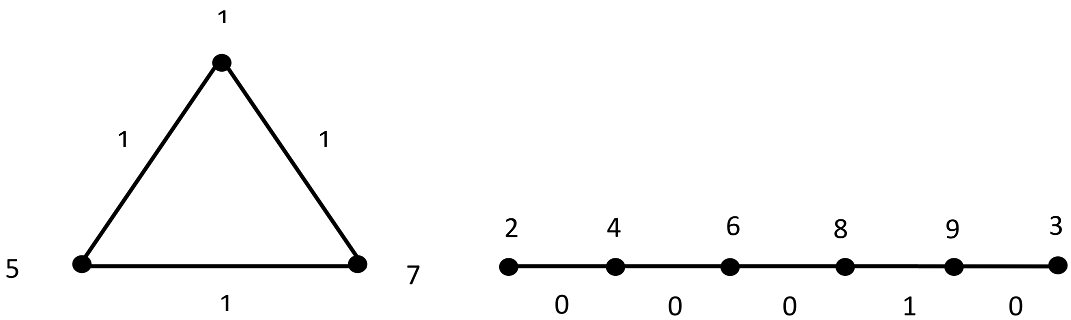

Figure 4: Disconnected graph $C_3 \cup P_6$.

3.3 Example:

The graph is given in figure 28 is $C_3 \cup P_7$. The order and size are 10 and 9.

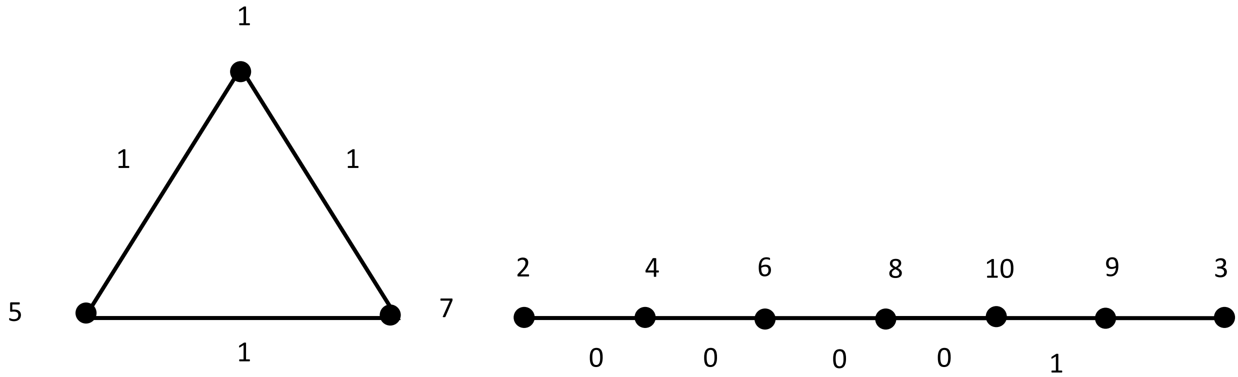

Figure 5: Disconnected graph $C_3 \cup P_7$.

3. 4 Example:

The graph is given in figure 29 is $C_5 \cup P_6$. The order and size are 11 and 10.

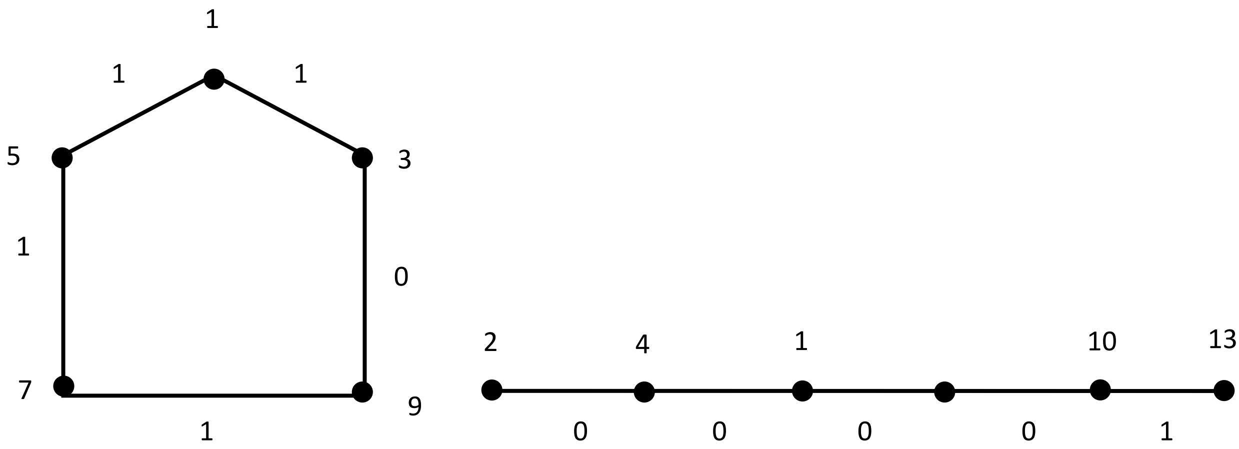

Figure 6: Disconnected graph $C_5 \cup P_6$.

3.5 Example:

The graph is given in figure 30 is $C_5 \cup P_7$. The order and size are 12 and 11.

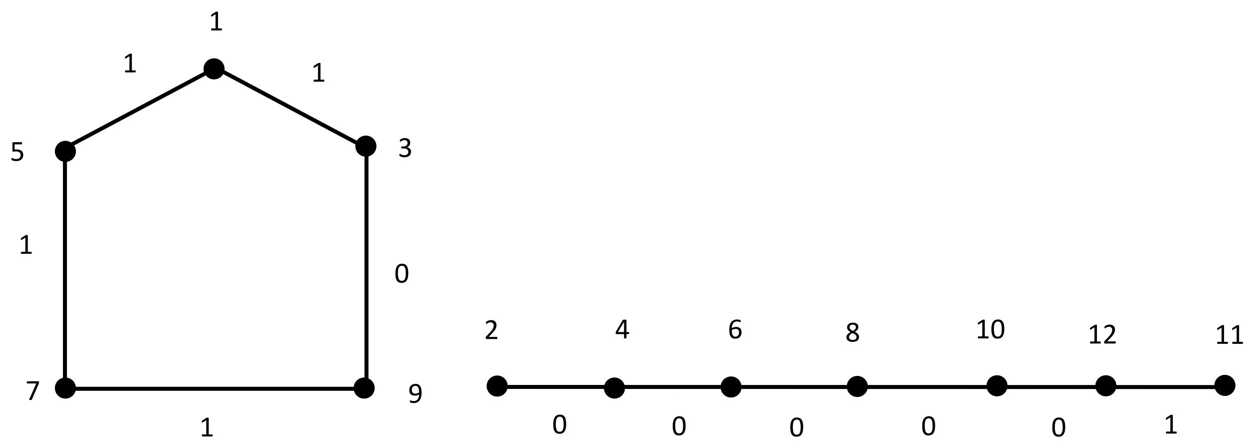

Figure 7: Disconnected graph $C_5 \cup P_7$.

4. Conclusion:

In this paper we proved cycle related graphs are prime cordial labeling.

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