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AN INTERESTING DIOPHANTINE PROBLEM ON TRIPLES-IV

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Abstract:

We search for three non-zero distinct integers a, b, c such that, if a non-zero integer is added to the sum of any pair of them as well as to their sum, the results are all squares.

Introduction:

The problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets we have studied by Diophantus [1]. A set of m positive integers $\{a_1, a_2, a_3, ...a_m\}$ is said to have the property D(n), $n \in Z - \{0\}$ if $a_i a_j + n$ is a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuple with property D(n).

Many mathematicians considered the construction of different formulations of Diophantine Triples with the property D(n) for any arbitrary integer n and also, for any linear polynomials in n. In this context, one may refer [2-22] for an extensive review of various problems on Diophantine Triples. In [23-26], the construction of special Dio–Triples, special Dio–Quadruples are considered and the special mention is provided because it differs from the earlier one and the special Dio-Triple (special Dio-Quadruple) is constructed where the product of any two numbers of the triple (quadruple) with addition of the same member and the addition with a non-zero integer or a polynomial with integer coefficients satisfies the required property.

This paper aims at constructing an interesting triple where, the sum of any two members of the set or their sum such that, if a non- zero integer is added to the sum of any pair of them as well as to their sum, the results are all squares.

Keywords: Diophantine Problem, Integer Triple & System of Equations.

Method of Analysis:

Let N be any given non-zero integer. Let a,b,c be three non-zero distinct integers such that

$$a + b = x^2 + 2Nx + N^2 - N \quad (1)$$

$$b + c = x^2 + 2(N+3)x + (N+3)^2 - N \quad (2)$$

$$a + b + c = x^2 + 2(N+6)x + (N+6)^2 - N \quad (3)$$

From (2) and (3), we get

$$a = 6x + 6N + 27 \quad (4)$$

From (1) and (3), we get

$$c = 12x + 12N + 36 \quad (5)$$

From (2), note that

$$b = x^2 + 2Nx - 6x + N^2 - 6N - 27 - N \quad (6)$$

It is noticed that, each of the expressions $a + b + N, b + c + N \,\&\, a + b + c + N$ is a perfect square.

Now,

$$a + c + N = 18x + 19N + 63 = y^2 \text{(say)} \tag{7}$$

Let (x_0, y_0) be any solution satisfying (7). Then, after a few calculations, it is found that the general values of x and y interms of $x_{0,} y_0$ are given by [27],

$$\begin{aligned} x_n &= x_0 + n^2 a - 2ny_0 \\ y_n &= (-1)^n (y_0 - na) \end{aligned} \tag{8}$$

Substituting the value of x in (4)-(6), we obtain the required triples, Note that, on employing (8), many triples are generated and it is worth mentioning that the values of x for which a, b and c are distinct are to be considered. A few examples of a, b and c for N=1, 7, 13, 25, 31 are presented in the below table:

Table:

N	y_n	n	x_n	a	b	c
1	$18k-10$	0	$18k^2-20k+1$	$108k^2-120k+39$	$(18k^2-20k+1)^2$ $-72k^2-80k-37$	$216k^2-240k+60$
		1	$18k^2-56k+39$	$108k^2-336k+267$	$(18k^2-56k+39)^2$ $-72k^2+224k-189$	$216k^2-672k+516$
		2	$18k^2-92k+113$	$108k^2-552k+711$	$(18k^2-92k+113)^2$ $-72k^2+368k-485$	$216k^2-1104k+1404$
		3	$18k^2-128k+223$	$108k^2-768k+1371$	$(18k^2-128k+223)^2$ $-72k^2+512k-8925$	$216k^2-1536k+2724$
1	$18k-8$	0	$18k^2-16k-1$	$108k^2-96k+27$	$(18k^2-16k-1)^2$ $-72k^2+64k-29$	$216k^2-192k+36$
		1	$18k^2-52k+31$	$108k^2-312k+219$	$(18k^2-52k+31)^2$ $-72k^2+208k-157$	$216k^2-624k+420$
		2	$18k^2-88k+103$	$108k^2-528k+651$	$(18k^2-88k+103)^2$ $-72k^2+352k-445$	$216k^2-1056k+1284$
		3	$18k^2-16k+209$	$108k^2-96k+1287$	$(18k^2-16k+209)^2$ $-72k^2+64k-869$	$216k^2-192k+2556$
		0	$18k^2-28k$	$108k^2-168k+69$	$(18k^2-28k)^2$ $+144k^2-224k-27$	$216k^2-336k+120$

7	$18k-14$	1	$18k^2-64k+46$	$108k^2-384k+345$	$(18k^2-64k+46)^2$ $+144k^2-512k+341$	$216k^2-768k+672$
		2	$18k^2-100k+128$	$108k^2-600k+837$	$(18k^2-100k+128)^2$ $+144k^2-800k+997$	$216k^2-1200k+1656$
		3	$18k^2-136k+246$	$108k^2-816k+1545$	$(18k^2-136k+246)^2$ $+144k^2-1088k+1941$	$216k^2-1632k+3072$
7	$18k-4$	0	$18k^2-8k-10$	$108k^2-48k+9$	$(18k^2-8k-10)^2$ $+144k^2-64k-107$	$216k^2-96k$
		1	$18k^2-44k+16$	$108k^2-264k+165$	$(18k^2-44k+16)^2$ $+144k^2-352k+101$	$216k^2-528k+312$
		2	$18k^2-80k+78$	$108k^2-480k+537$	$(18k^2-80k+78)^2$ $+144k^2-640k+597$	$216k^2-960k+1056$
		3	$18k^2-116k+176$	$108k^2-696k+1125$	$(18k^2-116k+176)^2$ $+144k^2-928k+1381$	$216k^2-1392k+2232$
13	$18k-16$	0	$18k^2-32k-3$	$108k^2-192k+87$	$(18k^2-32k-3)^2$ $+360k^2-640k-9$	$216k^2-384k+156$
		1	$18k^2-68k+47$	$108k^2-408k+38$	$(18k^2-68k+47)^2$ $+360k^2-1360k+99$	$216k^2-816k+756$
		2	$18k^2-104k+133$	$108k^2-624k+90$	$(18k^2-104k+133)^2$ $+360k^2-2080k+2711$	$216k^2-1248k+1788$
		3	$18k^2-140k+255$	$108k^2-840k+1635$	$(18k^2-140k+255)^2$ $+360k^2-2800k+5151$	$216k^2-1680k+3252$
13	$18k-2$	0	$18k^2-4k-17$	$108k^2-24k+3$	$(18k^2-4k-17)^2$ $+360k^2-80k-289$	$216k^2-48k-12$
		1	$18k^2-40k+5$	$108k^2-240k+135$	$(18k^2-40k+5)^2$ $+360k^2-800k+151$	$216k^2-480k+252$

		2	$18k^2 - 76k + 63$	$108k^2 - 456k + 483$	$(18k^2 - 76k + 63)^2 + 360k^2 - 1520k + 1311$	$216k^2 - 912k + 948$
		3	$18k^2 - 112k + 157$	$108k^2 - 672k + 1047$	$(18k^2 - 112k + 157)^2 + 360k^2 - 2240k + 3191$	$216k^2 - 1344k + 2076$
25	$18k - 14$	0	$18k^2 - 28k - 19$	$108k^2 - 168k + 63$	$(18k^2 - 28k - 19)^2 + 792k^2 - 1232k - 413$	$216k^2 - 336k + 108$
		1	$18k^2 - 64k + 27$	$108k^2 - 384k + 339$	$(18k^2 - 64k + 27)^2 + 792k^2 - 2816k + 1611$	$216k^2 - 768k + 660$
		2	$18k^2 - 100k + 109$	$108k^2 - 60k + 831$	$(18k^2 - 100k + 109)^2 + 792k^2 - 4400k + 423$	$216k^2 - 1200k + 1644$
		3	$18k^2 - 136k + 22$	$108k^2 - 861k + 1539$	$(18k^2 - 136k + 227)^2 + 792k^2 - 5984k + 10411$	$216k^2 - 1632k + 3060$
25	$18k - 4$	0	$18k^2 - 8k - 29$	$108k^2 - 48k + 3$	$(18k^2 - 8k - 29)^2 + 792k^2 - 352k - 853$	$216k^2 - 96k - 12$
		1	$18k^2 - 44k - 3$	$108k^2 - 264k + 159$	$(18k^2 - 44k - 3)^2 + 792k^2 - 1936k + 291$	$216k^2 - 528k + 300$
		2	$18k^2 - 80k + 59$	$108k^2 - 480k + 531$	$(18k^2 - 80k + 59)^2 + 792k^2 - 3520k + 3019$	$216k^2 - 960k + 1044$
		3	$18k^2 - 116k + 157$	$108k^2 - 696k + 1119$	$(18k^2 - 116k + 157)^2 + 792k^2 - 5104k + 7331$	$216k^2 - 1392k + 2220$
31	$18k - 16$	0	$18k^2 - 32k - 22$	$108k^2 - 192k + 81$	$(18k^2 - 32k - 22)^2 + 1008k^2 - 1792k - 515$	$216k^2 - 384k + 144$
		1	$18k^2 - 68k + 28$	$108k^2 - 408k + 381$	$(18k^2 - 68k + 28)^2 + 1008k^2 - 3808k + 2285$	$216k^2 + 816k + 744$
		2	$18k^2 - 104k + 114$	$108k^2 - 624k + 897$	$(18k^2 - 104k + 114)^2 + 1008k^2 - 5824k + 7101$	$216k^2 - 1248k + 1776$

		3	$18k^2 - 140k + 236$	$108k^2 - 840k + 1629$	$(18k^2 - 140k + 236)^2 + 1008k^2 - 7840k + 13933$	$216k^2 - 1680k + 3240$
31	$18k - 2$	0	$18k^2 - 4k - 36$	$108k^2 - 24k - 3$	$(18k^2 - 4k - 36)^2 + 1008k^2 - 224k - 1299$	$216k^2 - 48k - 24$
		1	$18k^2 - 40k - 14$	$108k^2 - 240k + 129$	$(18k^2 - 40k - 14)^2 + 1008k^2 - 2240k - 67$	$216k^2 - 480k + 240$
		2	$18k^2 - 76k + 44$	$108k^2 - 456k + 477$	$(18k^2 - 76k + 44)^2 + 1008k^2 - 4256k + 3181$	$216k^2 - 912k + 936$
		3	$18k^2 - 112k + 13$	$108k^2 - 672k + 1041$	$(18k^2 - 112k + 138)^2 + 1008k^2 - 6272k + 8445$	$216k^2 - 1344k + 2064$

Conclusion:

In this paper, we have considered an interesting Diophantine problem of constructing triples, which are such that, in each triple, the sum of any two as well as their sum, when added with a non-zero integer, represents a perfect square. The beauty of many Diophantine problems lies in the fact that they are neither trivial nor difficult to analyze. To conclude, one may investigate several further new explicit Diophantine problems.

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