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A STUDY ON RELATIONSHIP BETWEEN MATHEMATICS AND MUSIC

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Abstract:

Mathematics is a scientific study, full of order, countability and calculability. Music on the other hand, is thought to be artistic and expressive. The studies of these two disciplines, though seemingly different, however, are linked and have been for over two thousand years. Music itself is indeed very mathematical and mathematics is inherent to many basic ideas in music theory. Music theorists, like experts in other disciplines, use mathematics to develop, express and communicate their ideas. Mathematics can describe many phenomena and concepts in music. It explains how strings vibrate at certain frequencies, and sound waves are used to describe these mathematical frequencies. Modern technology used to make recordings on a compact disc (CD) or a digital video disc (DVD) also rely on mathematics. The relationship between mathematics and music is complex and constantly expanding. In this paper, we examine both the mathematics and music in the light of history. Further, we examine the relationship between Mathematics and Music by the performance of students in subject music and mathematics. It has been observed that there is a positive significant correlation between the two subjects. The results obtained in this study are in agreement with the earlier studies conducted across the globe.

Keywords: Music, Mathematics, Compact Disc, Digital Video & Correlation

Introduction:

Mathematics is a science of all sciences and an art of all arts. It has been a playback pioneer in scientific advancement of the modern civilization and is a subject that constantly changes. In the twenty-first century, a western view of mathematics is that it is the abstract science of shape, space, change, number, structure and quantity. It has been in existence since ancient civilizations and in some form has been approached, used and studied in different ways for thousands of years. The Inca, Egyptians and Babylonians all used it, yet mathematics was not studied for its own sake until Greek Antiquity (600-300 BC). Brilliant Greek academics contributed a wealth of knowledge about music, philosophy, biology, chemistry, physics, architecture and many other disciplines. Pythagoras, Plato and Aristotle were three very clever academics, and very influential figures when detailing the historic connection between mathematics and music. Pythagoras was born in the Classical Greek period (approximately 600 BC to 300 BC) when Greece was made of individual city-states. A dictator governed the island on which he lived, so he fled to Italy where he founded a religion (often called a cult) of mathematics. The followers of this religion known as Pythagoreans believed that mathematical structures were mystical. Pythagoreans had elaborate rituals and rules based on mathematical ideas. The numbers 1, 2, 3 and 4 were considered divine and sacred also they believed reality was constructed out of these numbers and numbers 1, 2, 3 and 4 were deemed the building blocks of life [Joseph, 1991]. Pythagoras attributed various numbers and forms to physical elements, for example, five is the cause of colour, six of cold, seven of health, and eight of love. According to Pythagoras, where harmony is, there are numbers. Plato who founded the first university in Greece was a Pythagorean who lived after the Golden Age of Ancient Greece (450 B.C approximately) and believed that mathematics was the core of education. Mathematics was so central to the curriculum, that above the doors of the university, the words "Let no man enter through these doors if ignorant of geometry" were written. It is recorded that many famous Greek mathematicians attended Plato's University. Aristotle a man of great genius, the teacher of Alexander the Great, is an example of a famous student of Plato. Aristotle together with Plato and Socrates (Plato's teacher) was one of the most important founding figures in western philosophy. Ancient Greek mathematics education was comprised of four sections: number theory, geometry, music and astronomy; this division of mathematics into four sub-topics is called a quadrivium [Papadopoulos, 2002]. The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this subject, but saturated with it, they fancied that the principles of mathematics were the principles of all things (Aristotle, 350 B.C., pgs. 1-5). The origin of mathematics is subject to much scholastic debate (Ernest, 2008). All available evidence suggests that the human species has had a recognizable concept of abstract numbers for at most 8,000 years (Field, H.H., 1980, etc).

Music theory is a beautiful subject that has been studied by people for thousands of years. Some of the oldest physical artifacts found in human and proto-human excavation sites are musical instruments: bone flutes and animal skins stretched over tree stumps to make drums. Whenever humans come together for any reason, music is there: weddings etc. In comparison to mathematics, music is defined as the art or science of combining vocal or instrumental (or both) sounds to produce beauty of form and harmony. Like mathematics, music has been an integral aspect of cultures throughout history (Nusrat and Bilal, 2014). Music is an artistic way of expressing emotions and ideas, and is often used to express and portray one's self and identity. Often music theorists study the language and notation of music. They seek to identify patterns and structures found in composer's techniques, across or within genres, and of historical periods. The four ways division of mathematics, which detailed music should be studied as part of mathematics, lasted until the end of the Middle Ages (approximately 1500 AD) in European culture. Theoretical music became an independent field, yet strong links with mathematics were maintained. Music research and teaching were occupations considered more prestigious than music composing or performing. This contrasts earlier times in history. Pythagoras, for example, was a geometer, number theorist and musicologist, but also a performer who played many different instruments.

Symbolic mathematics with equations proofs and theorems only dates back about 2,500 years. Calculus wasn't developed until the 17th century, while negative numbers were not in wide-spread use until the 18th century, and modern abstract algebra, where symbols like *x, y,* and *z* denote arbitrary entities, is just over 150 years old. In the 17th and 18th centuries, several of the most prominent and significant mathematicians were also music theorists [Papadopoulos, 2002]. Leibnitz a great mathematician writes, Music is a hidden exercise in arithmetic of a mind unconscious of dealing with numbers. René Descartes had many mathematical achievements include creating the field of analytic geometry, and developing Cartesian geometry. His first book, *Compendium Musicale* (1618) was about music theory. Marin Mersenne, a mathematician, philosopher and music theorist is often called the father of acoustics. He authored several treaties on music, including *Harmonicorum Libri* (1635) and *Traité de* *l'Harmonie Universelle* (1636). He also corresponded on the subject with many other important mathematicians including Descartes, Isaac Beekman and Constantijn Huygens. John Wallis, Leonhard Euler, Jean d'Alembert, Jean Philippe Rameau etc. Rameau one of the greatest French mathematician and music theorist said: "Music is a science which must have determined rules. These rules must be drawn from a principle which should be evident, and this principle cannot be known without the help of mathematics. In the 18th century, calculus became a tool, and was used in discussions on vibrating strings. Brook Taylor, who discovered the Taylor Series, found a differential equation representing the vibrations of a string based on initial conditions, and found a sine curve as a solution to this equation [Archibald]. Daniel Bernoulli (1700-1782) and Leonhard Euler (1707-1783), Swiss mathematicians, and Jean-Baptiste D'Alembert (1717-1783), a French mathematician, physicist, philosopher, and music theorist, were all prominent in the ensuing mathematical music debate. The anthropological record suggests that humans have possessed relatively the same brain structure for over 50,000 years. The current size of the human brain was reached even earlier, about half of a million years ago. Albert Einstein's brain could have hypothetically existed even in the Iron Age, but nothing that we would call mathematics existed in the Iron Age. This means that whatever features of our brains enable us to do mathematics must have evolved thousands of years before we had any mathematics (Devlin, 2000).

This paper aims to give an overview of the intricate relationship between mathematics and music by examining its different aspects. The history of the study of mathematics and music is intertwined, so it is only natural to begin this research paper by briefly outlining this relationship. Questions and problems arising in music theory have often been solved by investigations into mathematics and physics throughout history. There are many examples of composers who use mathematical techniques throughout their work. It is reported that music often has a religious connotation and message, and religious composers often use music to express their ideas and beliefs. Further, it is observed that religion, music and mathematics are strongly connected (Bilal et al., 2015).

Objectives:

The present study was carried out with the following objectives

(i) To discuss Mathematics and Music in the light of history.
(ii) To study the relationship between Mathematics and Music on the basis of the academic performance of the students.

Research Hypothesis:

Null-hypothesis (H_o): There is no significant correlation between the academic performance in mathematics and music subject

Methodology:

The present study pertains to relationship between mathematics and music in the light of history. The aim of the study is to increase the knowledge about the relationship between mathematics and music listening. Primary data was collected by using well designed interview cum questionnaire schedule keeping in view the literature available on the topic. 100 students having music subject were selected for the present study from the different colleges of Kashmir Valley using purposive and simple random sampling techniques. Secondary data was collected from secondary sources like books, journals, unpublished dissertation and latest information available from internet. The data obtained through questionnaire was analysed using appropriate mathematical and statistical tools. Statistical softwares SPSS and R were also used as a support.

Results and Discussion:

Mathematicians have been attracted to the study of music theory since the Ancient Greeks, because music theory and composition require an abstract way of thinking and contemplation. This method of thinking is similar to that required for pure mathematical thought [Papadopoulos, 2002]. Further, the connection between music and mathematics has in recent times been substantiated by modern imaging techniques that show which parts of the brain are active while carrying out various tasks. As it turns out the image patterns produced when professional musicians listen to music are extremely similar to those images produced when professional mathematicians work on a mathematical problem. Apparently, expert musicians and mathematicians use the same brain circuits in their respective professions (Devlin, 2000, p. 78). It is interesting to note the relation between mathematics, music and magic. The numbers play a vital role in magic theory (Bilal et al., 2015). People use this technique since generations, the simple procedure is explained in Table 1. The words are assigned values and instead of words we use numbers to explain the phenomenon. Astronomers in different parts of the globe use different combination of numbers for different purposes for writing amulet etc. (see Table 2-6). The basic Table 1 is used for practical purpose.

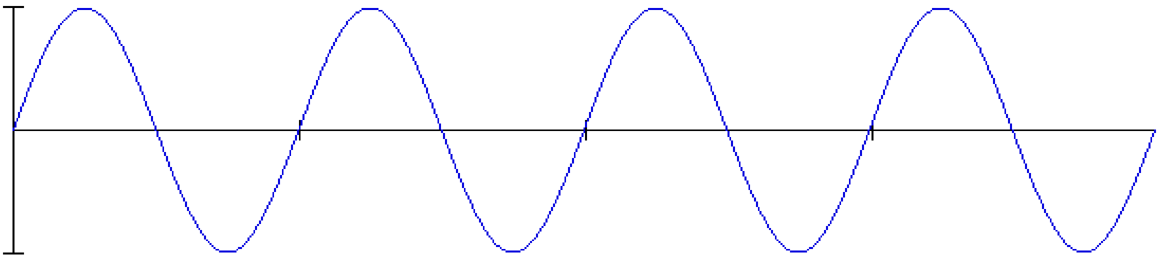
Table 1: Alphabet and their values

Alphabet	Value	Alphabet	Value	Alphabet	Value	Alphabet	Value
1. Alif ا	1	8. Ha ح	8	15. Seen س	60	22. Ta ث	400
2. Ba ب	2	9. Ta ط	9	16. Ain ع	70	23. Sa ث	500
3. Jeem ج	3	10. Ya ي	10	17. Fa ف	80	24. Kha خ	600
4. Dal د	4	11. Kef ك	20	18. Saad ص	90	25. Zai ز	700
5.Ha ه	5	12. Laam ل	30	19. Qaf ق	100	26.Zad ض	800
6. Wao و	6	13. Meem م	40	20. Ra ر	200	27. Za ظ	900
7. Za ز	7	14. Noon ن	50	21. Sheen ش	300	28. Ghain غ	1000

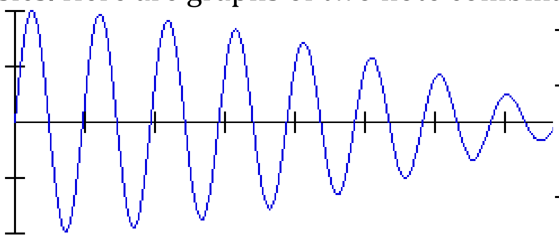
The inscription is written on paper or on bowl etc. In India the 3X3 magic square has been a part of rituals since Vedic times and is still today. The following Talisman (Taweez) Table 2, people use for beneficial purpose, although in Islam magic is not allowed to practice. In Islam it is told to go for extra prayer in case of any trouble. We give numerical values to the letters and obtain Table 3 and observe that it is 3 by 3 square and that sum of each row/column/diagonal is 15. On various transformations of Table 1, we obtain Table 4-6. The tables obtained by transformation are used in different ways and believed in astronomy to play role differently. In statistics, that is a branch of applied mathematics, we call this as 3X3 Latin Square Design (D.C. Montgomery, 2007). Numbers are used since centuries by astronomers and learned people for different purposes.

Table 2: Aulmet				Table 3: Numerical Form				Table 4: Interchanging Cell values				Table 5: Interchanging Cell values				Table 6: Interchanging Cell values		
ب ٢	ز ۷	و ٦		2	7	6		8	3	4		8	1	6		2	9	4
ط ٩	ه ٥	ا ١		9	5	1		1	5	9		3	5	7		7	5	3
د ٤	ج ٣	ح ٨		4	3	8		6	2	7		4	9	2		6	1	8

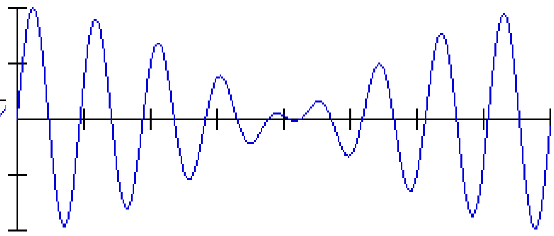
Every sound we hear is the consequence of pressure fluctuations traveling through the air and hitting our ear drums. The function that describes the audible pressure fluctuations of air is called a "sound wave". How the human ear processes sound is not completely known. Only when the vibration rate falls between 20 vibrations and 20,000 vibrations per second (20 Hz to 20,000 Hz) are we able to hear this sound energy. A crude mathematical model to understand how sounds travel through the air and what our ear does when it receives that sound is given by the differential equation $\partial^2 y/\partial^2 t = -Ky$, where t is the time and y is the distance of that point on the membrane from its equilibrium solution. The solutions to this equation give us the basic building blocks to understand all sounds. We know solutions to the differential equations are sine and cosine and the wave patterns of the periodic functions sine and cosine lend themselves perfectly as a model for describing the cyclical nature of vibrational energy, sound and music. Since the sine function $y = \sin t$ begins at the origin (when $t = 0$), sine is the more convenient of the two for this purpose. Here is a graph of a single note:



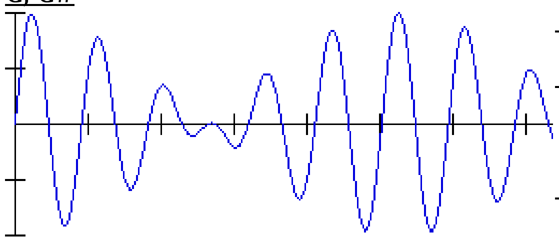
As an example, note A above middle C is the note on which most tunings of instruments is based (440 Hz). Musical tones are restricted to a limited set of frequencies we call notes. A song is a sequence of notes. The notes represented by $e^{-2t}\sin(2\pi.440t)$ and $e^{-2t}\sin(2\pi.880t)$ are an octave apart. The frequencies of the notes represented by $e^{-2t}\sin(2\pi.440t)$ and $e^{-2t}\sin(2\pi.660t)$ are consonant (pleasant sound) while $e^{-2t}\sin(2\pi.440t)$ and $e^{-2t}\sin(2\pi.450t)$ are dissonant (not pleasing to the ear). How we judge combinations of notes is partly subjective, but most people would agree on some simple basics. Here are graphs of two note combinations.



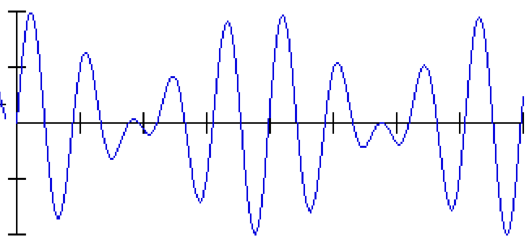
<u>C, C#</u>



<u>C, D</u>



<u>C, D#</u>



<u>C, E</u>

It is reported that those who create music use symbolic language as well as a rich system of notation, including diagrams similar to mathematical graphs of discrete functions in two dimensional Cartesian coordinates. At the beginning of a piece of music, after the clef is marked, the time signature is marked by a fraction on the music staff. Common time signatures include 2/4, 3/4, 4/4. and 6/8. The denominator of the fraction, is the unit of measure, and used to denote pulse. The numerator indicates the number of these units or their equivalent included in the division of a measure. Groups of stressed and relaxed pulses in music are called meters. The meter is also given in the numerator of the time signature [Olson and Harry, 1967]. Common meters are 2, 3, 4, 6, 9, 12 which denote the number of beats or pulses in the measure. For example, take the time signature 3/4. Each measure is equivalent to three (information from the numerator) quarter notes (information from the denominator). The count in each measure would be: 1, 2, 3. The 1 is the stressed pulse, while the 2 and 3 are relaxed. The time signature 3/4 is common in waltzes [Olson and Harry, 1967]. Besides abstract language and notation, mathematical concepts such as symmetry, periodicity, proportion, discreteness, and continuity make up a piece of music. Numbers are also very instrumental, and influence the length of a musical interval, rhythm, duration, tempo and several other notations.

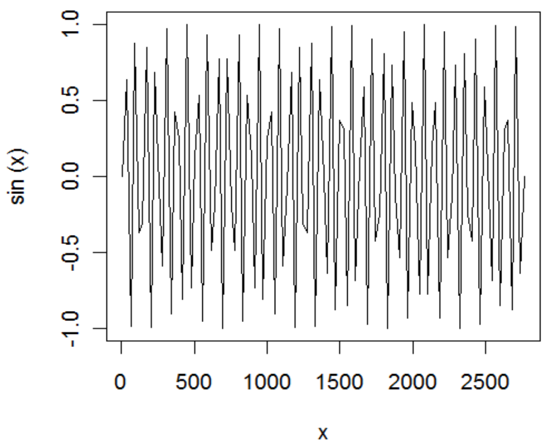


Fig 1: sin(2* π*440t)

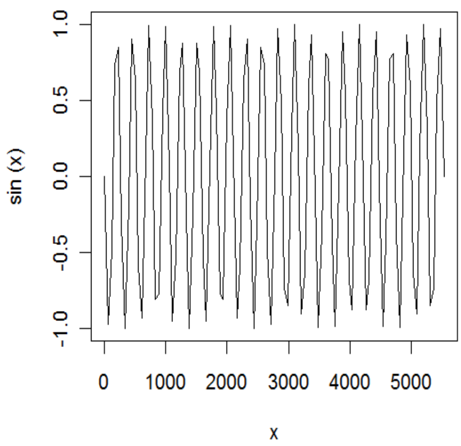


Fig 2: sin(2* π *880t)

Figure 3: The shape of a simply plucked string based on Euler's ideas.

Figure 1 and 2 present two different graphs and produce different sounds. Figure 3 as given by Euler gives idea that a simply plucked string at starting position represents two lines, which cannot be represented in one equation as given by D'Alembert. The wave equation is a differential equation that examines the behaviour of a piece of a string based on initial condition, displacement, and release from rest. Fourier analysis is used to identify naturally occurring harmonics, to model sound, and to break up sound into pieces that define it. We use R software to generate the above graphs. It has been observed that there is a significant positive correlation between mathematics and music subject on the basis of academic performance of students ($p<0.01$). Further, results obtained in this study revealed that academic performance of students is significantly correlated with factors like Socio-economic status, Gender, Parental education, Residence of an individual.

The two fields have been studied in such unison, that musical words have been applied to mathematics. For example, harmonic is a word that is used throughout mathematics (harmonic series, harmonic analysis), yet its origin is in music theory. It's been discussed that throughout history, mathematicians have long been fascinated with music theory. The study of these two disciplines, though seemingly different, however, is linked and has been for over two thousand years. Music itself is indeed very mathematical, and mathematics is inherent to many basic ideas in music theory. Music theorists, like experts in other disciplines, use mathematics to develop, express and communicate their ideas. Every musical instrument has capabilities and limitations that help in distinguishing one instrument from the other. Scientists are interested in sound synthesis and sound analysis studies. Statistical techniques are used in composition (Jones, 1981) and in forecasting unfinished works (Dirst and Weigend, 1992). Instruments are mathematical; cellos have a particular shape to resonate with their strings in a mathematical fashion. Modern technology used to make recordings on a compact disc (CD) or a digital video disc (DVD) also relies on mathematics. The relationship between mathematics and music is complex and constantly expanding.

Conclusion:

In this paper, we discuss how mathematics and music are related to each other. The relationship between mathematics and music is immense. It spans over two thousand years of history, and involves hundreds of people ranging from mathematicians, to musicians, to music theorists. Beauty of a piece of art depends on the manner in which it expresses truth. Mathematics is knowledge of truth and realities. It is itself a piece of fine art. It is a thing of beauty, and for many people it is a joy forever especially when they do not study it for examination purpose. Everybody cannot appreciate fully a beautiful piece of architecture, a painting of musical notes. Only a mathematical mind can appreciate these arts with some sense of confidence. Appreciation of rhythm, proportion, balance and symmetry postulates a mathematical mind. Further, we discussed in historical context the beginning of relationship between Mathematics and Music in view of the contributors to this field namely Pythagoras, J.P. Rameau, Fibonacci's, Messiaen, Bach etc. Research and literature has been published on the different characters, eras and contributions involved. It would be impossible to discuss every aspect of this complex relationship in this paper. This paper has thus provided a "snap shot" of this relationship. We have tried to include the topics we found most interesting, and that we could best relate to and understand given our mathematics and music knowledge. Individuals vary in their views on which connections between mathematics and music are valid, and which are most consequential and significant. We have discussed the main events and people, who we think have made a great contribution to this field, all the while striving to give as broad an overview of this subject as possible. Finally, the relationship between Mathematics and Music on the basis of the academic performance of students in subject music and mathematics was examined. The result showed Null-hypothesis (Ho) is rejected. The study found that there is a significant relationship in academic performance of students between the two subjects . The results obtained in this study are in agreement with the earlier studies conducted across the globe. It seems musicians do not show the same interest for mathematics as mathematicians show for music but mathematical thinking, mind-setting and problem solving approach is related. Several studies have pursued this approach, and the results have been mixed. The relationship between mathematics and music is incredibly interesting, and this exploration is one that could last a lifetime!

Limitations of Present Study:

(a) The limitations of this study are similar to the limitations of knowledge in general. Since this study involves asking the question of "precisely what this mathematical knowledge is and where it comes from," it is limited by the limits of reason itself so to speak.

(b) Sample size is very less as compared to other subjects very few students choose music as subject to study.

(c) Mathematics has different branches so it is better to study relationship between music and different branches of mathematics separately.

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