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NUMERICAL STABILITY METHOD IN NUMERICAL ANALYSIS

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Abstract:

Numerical analysis naturally finds applications in the fields of Engineering and the physical science, but in the 21st century, the life Sciences and even the arts have adopted elements of scientific computation. Ordinary differential equations appear in the movement of heavenly Bodies (planets, Stars and galaxies) optimization occurs in portfolio Management numerical linear algebra is important for data analysis stochastic differential equations and Markov chains are essential in simulating living cells for medicine and biology.

Keywords: Babylonian Method, Numerical Stability & Loss of Significance.

Introduction:

Numerical analysis is the study of algorithms that use numerical approximation for the problems of mathematical analysis.

Numerical stability:

Numerical stability is an important notion in numerical analysis. An algorithm is called numerically stable if an error, what ever its cause does not grow to be much larger during the calculation. This happens if the problem is well conditioned meaning that the solution changes by only a small amount if the problem data are changed by a small amount. To the contrary if a problem is ill-conditioned then any small error in the data will grow to be a large error.

Both the original problem and the algorithm used to solve that problem can be well conditioned and / or ill conditioned, and any combination is possible.

So an algorithm that solves a well-conditioned problem may be either numerically stable or numerically unstable. An art of numerical analysis is to find a stable algorithm problem for instance computing the square root of 2 (which is roughly 1.41421) is a well-posed problem [1] by starting with an initial approximation X_1 to $\sqrt{2}$, for instance $X_1=1.4$ and then computing improved guesses X_2, X_3, etc., one such method is the famous Babylonian method, which is given by $X_{k+1} + 1/X_k$ Another iteration, which we will call method x is given by $X_{k+1} = (X_k^2-2)^2 + X_k^{(3)}$. We have calculated a few iteration of each scheme in table from below with initial guesses $X_1=1.4$ and $X_1= 1.42$.

Example:

$f(x) = \sqrt{x}$ is continuous and so evaluating it is well-posed at least for x being not close to zero.

Babylonian	Babylonian	Method x	Method x
$X_1 =1.4$	$X_1 = 1.42$	$X_1 = 1.4$	$X_1 = 1.42$
$X_2 = 1.4142857$	$X_2 = 1.41422535$	$X_2 = 1.4016$	$X_2 = 1.42026896$
$X_3 = 1.414213564$	$X_3 = 1.41421356242$	$X_3 = 1.4028614$	$X_3 = 1.42056$
		$X_{1000000} = 1.41421$	$X_{28} = 7280.2284..$

Observe that the Babylonian method converges fast regardless of the initial guesses, where as method x converges extremely slowly with initial guess1.4. Hence, the Babylonian method is numerically stable, while method x is numerically unstable.

Numerical stability is affected by the number of the significant digits the machine deeps on if we use a machine that keeps on the first four floating point digits; a good example on loss of significance is given by these two equivalent functions.

Example:

$F(x) = x(\sqrt{x+1} - \sqrt{x})$ and

$g(x) = x/\sqrt{x+1}+\sqrt{x}$

If we compare the results of

$F(500) = 500(\sqrt{501} - \sqrt{500})$
$=500(22.3830 - 22.3607)$
$=500(0.0223)$
$=11.1500$ and

$g(500) = 500/(\sqrt{501} + \sqrt{500})$
$= 500/(22.3830 + 22.3607)$
$= 500/44.7437$
$= 11.1748$

By looking to the two above results we realize that loss of significance [2] which is also called subtractive cancellation has a huge effect on the results, even though both functions are equivalent; to show that thy are equivalent simply we need to start by f(x) and end with g(x) and so,

$$F(x) = x(\sqrt{x+1} - \sqrt{x})$$
$$= \frac{x(\sqrt{x+1} - \sqrt{x})(\sqrt{x+1} + \sqrt{x})}{(\sqrt{x+1} + \sqrt{x})}$$
$$= \frac{x(\sqrt{x+1})^2 - (\sqrt{x})^2}{(\sqrt{x+1} + \sqrt{x})}$$
$$= \frac{x}{(\sqrt{x+1} + \sqrt{x})}$$

The true value for the result is 11.174755..., which is exactly g(500)=11.1748. After rounding the result to 4 decimal digits. .

Conclusion:

In this paper the Numerical stability is used to solve that problem can be well conditioned and / or ill conditioned, and any combination is possible.

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