International Journal of Multidisciplinary Research and Modern Education (IJMRME)
ISSN (Online): 2454 - 6119
(www.rdmodernresearch.org) Volume I, Issue I, 2015

ON THE TERNARY CUBIC EQUATION

$$5(x^2+y^2)-8xy=74(k^2+s^2)z^3$$

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Abstract:

The ternary cubic Diophantine equation given by $5(x^2+y^2)-8xy=74(k^2+s^2)z^3$ *is analyzed for its non-zero distinct integer points on it. Different patterns of integer points for the equation under consideration are obtained. A few interesting relations between solutions and special numbers are obtained.*

Keywords: Ternary Cubic, Integer Solutions & Polygonal Number

Introduction:

The ternary cubic diophantine equation offers an unlimited field of research due to their variety [1-3]. For an extensive of various problems, one may refer [4-9]. This communication concerns with yet another interesting ternary cubic diophantine equation $5(x^2+y^2)-8xy=74(k^2+s^2)z^3$ for determining its infinitely many non-zero integral points. Also, a few interesting relations between the solutions and special numbers are presented.

Notations:

1) Polygonal Number of rank 'n' with size m:

$$t_{m,n}=n[1+\frac{(n-1)(n-2)}{2}]$$

2) Centered Pyramidal Number of rank 'n' with size m:

$$cp_{m,n}=[\frac{m(n-1)n(n+1)}{6}]+n$$

Method of Analysis:

The Diophantine equation to be solved for its non-zero distinct integral solution is

$$5(x^2+y^2)-8xy=74(k^2+s^2)z^3 \quad (1)$$

The substitution of linear transformations,

$$x=u+v, y=u-v, u\neq v\neq 0 \quad (2)$$

in (1) leads to $u^2+9v^2=37(k^2+s^2)z^3$ (3)

Let $z=a^2+9b^2$ (4)

Where a and b are non-zero distinct integers. Different patterns of integer solutions to (1) are illustrated below.

Method 1:

Write 37 as $37=(6+i)(6-i)$ (5)

Substituting (4), (5) in (3) and using the method of factorization, we get

$$(u+i3v)(u-i3v)=(6+i)(6-i)(k+is)(k-is)(a+i3b)^3(a-i3b)^3$$

Employing positive and negative factors, we get

$$(u+i3v)=(6+i)(k+is)(a+i3b)^3 \quad (6)$$

$$(u+i3v)=(6-i)(k-is)(a-i3b)^3 \quad (7)$$

Equating real and imaginary parts in (6)

$$u=(6k-s)(a^3-27ab^2)-(6s+k)(9a^2b-27b^3)$$
$$v=\frac{1}{3}[(6k-s)(9a^2b-27b^3)+(6s+k)(a^3-27ab^2)] \quad (*)$$

As our interest is on finding integer solutions, we choose a and b suitably so that the values of u and v are in integers.

Replacing by 3A and b by 3B in (*), (4) and in view of (2), the corresponding integer solution to (1) are

$$u=(6k-s)(27A^3-729AB^2)-(6s+k)(243A^2B-729B^3)$$
$$v=(6k-s)(81A^2B-243B^3)+(6s+k)(9A^3-243AB^2)$$
$$z=9A^2+81B^2$$

In view of (2), we get

$$x=x(A,B,k,s)=k(171A^3-4617AB^2+243A^2B-729B^3)+ s(27A^3-729AB^2-1539A^2B+4617B^3)$$
$$y=y(A,B,k,s)=k(153A^3-4131AB^2-729A^2B-2187B^3)+ s(-81A^3+2187AB^2-1377A^2B+4131B^3)$$
$$z=z(A,B)=9A^2+81B^2$$

For simplicity and clear understanding we present below the solutions and the corresponding properties of (1) when k=1, s=4.

Thus the integer solutions of the equations $5(x^2+y^2)-8xy=74(k^2+s^2)z^3$ are given by

$$x(A,B,1,4)=279A^3-17739AB^2-7533A^2B-5913B^3)$$
$$y(A,B,1,4)=-171A^3-4617AB^2-6237A^2B-18711B^3)$$
$$z(A,B)=9A^2+81B^2$$

Properties:

1) $42[y(1,1,1,4)-x(1,1,1,4)]$ is a nasty number
2) $x(a,a,1,4)-y(a,a,1,4)+12348cp_{6,a}=0$
3) $7\{y(1,1,1,4)-x(1,1,1,4)\}$ is a perfect square
4) $6\{y(1,1,1,4)-x(1,1,1,4)\}$ is a cubical integer
5) $x(b,b,1,4)-y(b,b,1,4)-12348cp_{6,b}=0$
6) $x(a,1,1,4)-279cp_{6,a}+5913t_{4,a}-17739\equiv 0(\text{mod}\,7533)$

Method 2:

Instead of (5), 37 can be written as $37=(1+6i)(1-6i)$ (**)

Preceding as in method I, the non-zero distinct integral solutions to (1) are given by

$$x=x(A,B,k,s)=k(81A^3-2187AB^2-1377A^2B+4131B^3)+ s(-153A^3-4131AB^2-729A^2B+2187B^3)$$
$$y=y(A,B,k,s)=k(-27A^3+729AB^2-1539A^2B+4617B^3)+ s(-171A^3+4617AB^2+243A^2B-729B^3)$$
$$z=z(A,B)=9A^2+81B^2$$

For simplicity and clear understanding we present below the solutions and the corresponding properties of (1) when k=2, s=1.
Thus, the integer solutions of the equation $5(x^2+y^2)-8xy=370z^3$ are given by

$$x = x(A,B,2,1) = 9A^3 - 243AB^2 - 3483A^2B + 10449B^3$$
$$y = y(A,B,2,1) = -225A^3 + 6075AB^2 - 2835A^2B - 8505B^3$$
$$z = z(A,B) = 9A^2 + 81B^2$$

Properties:

$798[y(a,a,2,1) - x(a,a,2,1)]$ is a nasty number

$x(a,a,2,1) - y(a,a,2,1) + 4788cp_{6,a} = 0$

$133\{y(a,a,2,1) - x(a,a,2,1)\}$ is a perfect square

$x(a,1,2,1) + 3483t_{4,a} - 9cp_{6,a} - 10449 \equiv 0 \pmod{243}$

$y(b,b,2,1) - x(b,b,2,1) - 4788cp_{6,a} = 0$

$y(a,1,2,1) + 2835t_{4,a} + 225cp_{6,a} - 8505 \equiv 0 \pmod{6075}$

In addition to (5) and (**), 37 may also be represented as

$37 = (-6+i)(-6-i)$

$37 = (-1+6i)(-1-6i)$

Proceeding as in method 1, two moves choices of integer solutions to (1) are obtained.

Conclusion:

In this paper, we have presented different patterns of non-zero distinct integer solutions to the ternary cubic Diophantine equation considered. To conclude one may search for other patterns of integer solutions to (1) along with their properties.

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