



CALCULATING THE EXPECTED TIME TO CONTROL THE RECRUITMENT WHEN SYSTEM FAILS

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Abstract:

This paper describes the development of recruitment control to a time dependent and promotion period. Current manpower position to its desired position. Through planning, management strives to have the right number and the right kinds of people, at the right places, at the right time, doing things which result in both the organization and the individual in it. Three parameter generalized Pareto distribution is used to compute the expected time to reach the recruitment status of the organization and its variance is found. The analytical results are numerically illustrated by assuming specific distributions.

Key Words: Three Parameter Generalized Pareto Distribution, Expected Time, Recruitment, Promotion Period & Manpower.

Introduction and Motivation:

Nowadays, it is generally accepted that the use of statistical models for manpower planning is well established. In today's business world, capacities are predominately determined by the available manpower resources. The generalized Pareto (GP) distribution was introduced by Pickands (1975) and has since been applied to a number of areas including socio-economic phenomena, physical and biological processes the GP distribution suitable for modelling flood magnitudes exceeding a fixed threshold.

Since Pickands (1975), it has been well known that the conditional distribution of any random variable over a high threshold is approximately generalized Pareto distribution (GPD), which includes the Pareto distribution, the exponential distribution and distributions with bounded support. Generalized Pareto distributions (GPD) are widely used for modelling excesses over high thresholds. In general, GPD can be applied to any situation in which the exponential distribution might be used but in which some robustness is required against heavier tailed or lighter tailed alternatives. One can see for more detail in Esary *et.al.*, (1973), Sathiyamoorthi (1980), Pandiyan *et.al.*, (2010), Jeeva *et.al.*, (2004) and Kannadasan *et.al.*, (2013) about the expected time to cross the threshold level of the organization. Mathematical model is obtained for the expected time of breakdown point to reach the threshold level through Three parameter generalized Pareto distributions

Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made. The exit of every person from the organization results in a random amount of depletion of manpower (in man hours). The process of depletion is linear and cumulative. The inter arrival times between successive occasions of wastage are i.i.d. random variables. If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable. The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

Notations:

X_1 : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i = 1, 2, 3, \dots, k$ and X_i 's are i.i.d and $X_i = X$ for all i .

Y : a continuous random variable denoting the threshold level having three parameter generalized Pareto distribution.

$g(.)$: The probability density functions (p.d.f) of X_i .

$g_k(.)$: The k - fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^k X_i$

$g * (.)$: Laplace transform of $g(.)$

$g_k^*(.)$: Laplace transform of $g_k(.)$

$h(.)$: The probability density functions of random threshold level which has three parameter generalized Pareto distribution and $H(.)$ is the corresponding Probability generating function.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding Probability generating function.

$V_k(t) : F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in $(0, t]$

$S(.)$: The survivor function i.e. $P[T > t]; 1 - S(t) = L(t)$.

Model Descriptions and Solutions:

The three-parameter generalized Pareto (GP) distribution. Let Y be the random variable which can be expressed as when $\alpha = 1$

$$F(x) = 1 - e^{\left(\frac{d-x}{b}\right)}, \quad a = 0$$

Where d is a location parameter, b is a scale parameter, a is a shape parameter. We find the survival function as

$$\bar{H}(x) = e^{\left(\frac{d-x}{b}\right)} \tag{1}$$

We consider a system that is subject to shocks, where each shock reduces the effectiveness of the system and makes it more expensive to run. Assume that shocks occur randomly in time in accordance with a three-parameter generalized Pareto (GP) distribution.

$P(X_i < Y) = [\text{The system does not fail, after } k \text{ epochs of exits}]$

$$P(X_i < Y) = \int_0^\infty g_k(x) \bar{H}(x) dx = \left[g^* \left(\frac{1-d}{b} \right) \right]^k \tag{2}$$

$$P(T > t) = \sum_{k=0}^\infty V_k(t) P(X_i < Y)$$

It is also known from renewal process that

$$P(\text{exactly } k \text{ policy decisions in } (0, t]) = F_k(t) - F_{k+1}(t) \quad \text{with } F_0(t) = 1$$

$$= 1 - \left\{ \sum_{k=0}^\infty [F_k(t) - F_{k+1}(t)] \left[g^* \left(\frac{1-d}{b} \right) \right]^k \right\} \tag{3}$$

Now, $L(T) = 1 - S(t)$

Taking Laplace transform of $L(T)$, we get

$$= 1 - 1 + \left[1 - g^* \left(\frac{1-d}{b} \right) \right] \sum_{k=1}^\infty F_k(t) \left[g^* \left(\frac{1-d}{b} \right) \right]^{k-1} \tag{4}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c . Now $f^*(s) = \left(\frac{c}{c+s} \right)$, substituting in the above equation (4) we get

$$l^*(s) = \frac{\left[1 - g^* \left(\frac{1-d}{b} \right) \right] f^*(s)}{\left[1 - g^* \left(\frac{1-d}{b} \right) f^*(s) \right]} = \frac{\left[1 - g^* \left(\frac{1-d}{b} \right) \right] c}{\left[c + s - g^* \left(\frac{1-d}{b} \right) c \right]} \text{ on simplification} \tag{5}$$

$$E(T) = - \frac{d}{ds} l^*(s) \text{ given } S = 0$$

$$E(T) = \frac{1}{c \left[1 - g \left(\frac{1-d}{b} \right) \right]^{b+1-d}} \tag{6}$$

$$E(T^2) = \frac{2[b+1-d]^2}{c^2 [2b+1-d]^2} \tag{7}$$

Variance can be obtained

$$V(T) = E(T^2) - [E(T)]^2 = \frac{2[b+1-d]^2}{c^2 [2b+1-d]^2} - \left[\frac{b+1-d}{c [2b+1-d]} \right]^2$$

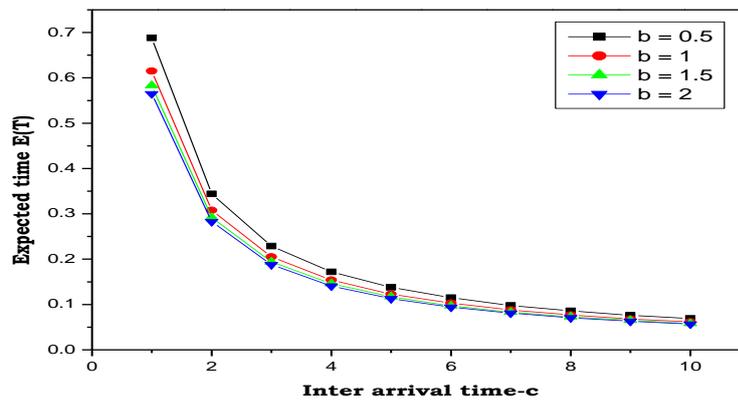
On simplification we get,

$$V(T) = \frac{[b+1-d]^2}{c^2 [2b+1-d]^2} \tag{8}$$

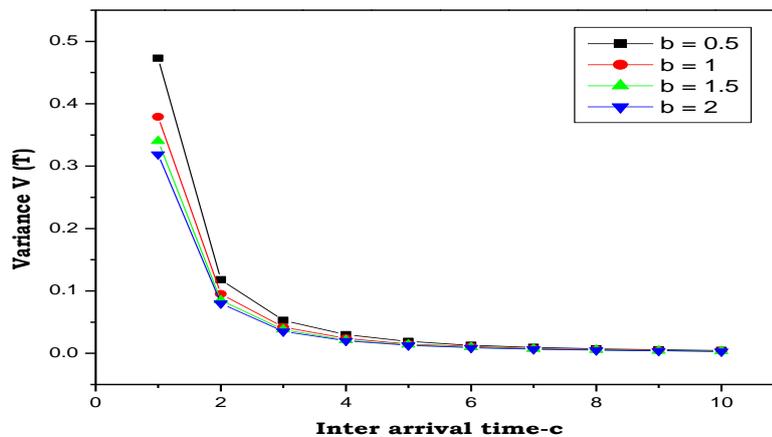
Numerical Illustration:

The mathematical models have been discussed by various authors taking into consideration, many hypothetical assumptions. Such models provide the possible clues relating to the consequences of infections, the time taken for recruitment etc.,

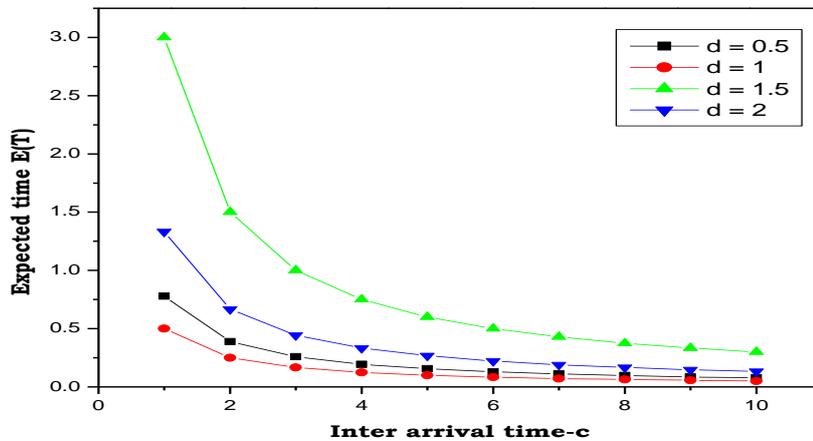
E (T) Figure (a)



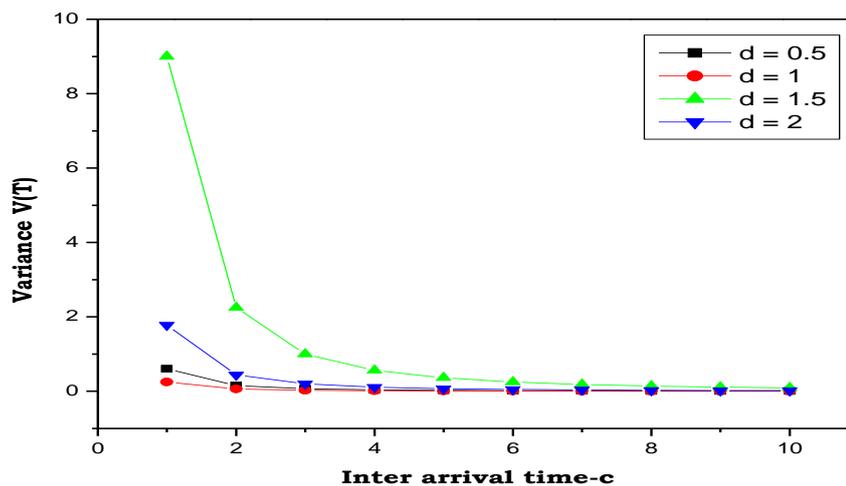
V (T) Figure (b)



E (T) Figure (a)



V (T) Figure (b)



Conclusions:

When b is kept fixed with other parameters d the inter-arrival time ' c ', which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E(T)$ to cross the threshold is decreasing, for all cases of the parameter value $b = 0.5, 1, 1.5, 2$. When the value of the parameter b increases, the expected time is found decreasing, this is observed in Figure 1a. The same case is found in variance $V(T)$ which is observed in Figure 1b.

When d is kept fixed with other parameters b the inter-arrival time ' c ' increases, the value of the expected time $E(T)$ to cross the threshold is found to be decreasing, in all the cases of the parameter value $d = 0.5, 1, 1.5, 2$. When the value of the parameter d increases, the expected time is found decreasing. This is indicated in Figure 2a. The same case is observed in the variance $V(T)$ which is observed in Figure 2b.

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