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QUINTIC WITH THREE UNKNOWNS

$$3(x^2+y^2)-2xy+2(x+y)+1=33z^5$$

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Abstract:

The ternary quintic Diophantine equation is analyzed for its infinitely many non-zero distinct integral solutions. A few interesting properties among the solutions are presented.

Keywords: Non - Homogeneous Quintic, Quintic With Three Unknowns & Integral Solutions

1. Introduction:

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular quantic equations, homogeneous or non-homogeneous, have aroused the interest of numerous mathematicians since antiquity [1-3]. For illustration, one may refer [4-8] for Quintic equation with three unknowns. This paper concerns with yet another the problem of determining non-trivial integral solutions of the non-homogeneous Quintic equation with three unknowns given by $3(x^2+y^2)-2xy+2(x+y)+1=33z^5$. A few relations among the solutions are presented.

2. Method of Analysis:

The ternary quintic Diophantine equation to be solved for its non-zero distinct integral solutions is $3(x^2+y^2)-2xy+2(x+y)+1=33z^5$ (1)

The substitution of linear transformations

$$x=u+v,\ y=u-v \qquad (2)$$

in (1) leads to $p^2+8v^2=33z^5$ (3)

where $p=2u+1$ (3A)

Let $z=a^2+8b^2$ (4)

where a and b are non-zero distinct integers

Different patterns of integral solutions to (1) are illustrated below

Pattern-1

Write 33 as $33=(1+i2\sqrt{8})(1-i2\sqrt{8})$ (5)

Substituting (4) & (5) in (3) and using the method of factorization, we get

$$(p+i\sqrt{8}v)(p-i\sqrt{8}v)=(1+i2\sqrt{8})(1-i2\sqrt{8})(a+i\sqrt{8}b)^5(a-i\sqrt{8}b)^5$$

Employing positive and negative factors, we get

$$(p+i\sqrt{8}v)=(1+i2\sqrt{8})(a+i\sqrt{8}b)^5 \qquad (6)$$

$$(p-i\sqrt{8}v)=(1-i2\sqrt{8})(a-i\sqrt{8}b)^5 \qquad (7)$$

Equating real and imaginary parts in (6)

$$p=a^5-80a^4b-80a^3b^2+1280a^2b^3+320ab^4-1024b^5$$

$$v=2a^5+5a^4b-160a^3b^2-80a^2b^3+640ab^4+64b^5$$

Using (3A) and Employing (2), we get

$$x(a,b)=x=\frac{5a^5-1}{2}-35a^4b-200a^3b^2+560a^2b^3+800ab^4-448b^5 \quad (8)$$

$$y(a,b)=y=\frac{-3a^5-1}{2}-45a^4b+120a^3b^2+720a^2b^3-480ab^4-576b^5 \quad (9)$$

$$z(a,b)=z=a^2+8b^2 \quad (10)$$

Choosing a to be odd in the above equations (8)-(10), one obtains the corresponding integer solutions to (1)

A few examples are presented below:

A	b	x	y	z
1	1	679	-263	9
	2	2076	-19964	33
3	1	-636	3694	17
	4	381075	-506849	137

Some interesting properties are presented below:

[*] $z(p^3-24pq^2, 3p^2q-8q^3)$ is a cubical integer

[*] If (x_0, y_0, z_0) is any given integer solutions of (1), then the triple $(-y_0-1, -x_0-1, z_0)$ also satisfies (1)

[*] For simplicity and clear understanding a few interesting relations among the solutions are presented when $a=1$

[i] $x(1,b)-y(1,b)+40z(1,b)\equiv 0(\text{mod}\,2)$

[ii] $15(6b+1)z(1,b)-9b[z(1,b)-1]^2-y(1,b)\equiv 17(\text{mod}\,135)$

[iii] $15(6b+1)[z(1,b)-1]-9b[z(1,b)-1]^2-y(1,b)\equiv 2(\text{mod}\,45)$

[iv] For the value of b given by $b=100\alpha^3-180\alpha^2+108\alpha-22$
$x(1,b)-y(1,b)+2[z(1,b)-1]^2(b-10)+20[z(1,b)-1](b+2)$ is a cubical integer

[v] For the value of b given by $b=10\alpha^2-16\alpha+6 \,\&\, b=10\alpha^2-4\alpha$
$x(1,b)-y(1,b)+2[z(1,b)-1]^2(b-10)+20[z(1,b)-1](b+2)$ is a perfect square

Note:

In addition to (5), 33 may also be represented by

$$33=(5+i\sqrt{8})(5-i\sqrt{8})$$

$$33=\frac{(17+i\sqrt{8})(17-i\sqrt{8})}{9} \quad (*)$$

Proceeding as in Pattern-1, two more choices of integer solutions to (1) are obtained.

Pattern-2

Rewrite (3) as $p^2+8v^2=33z^2*1$ (11)

Assume $1=\frac{(1+i\sqrt{8})(1+i\sqrt{8})}{9}, 33=(-1+i2\sqrt{8})(-1-i2\sqrt{8})$ (12)

Following the analysis presented in pattern-1, the corresponding integer solutions to (1) are found to be

$$x=\frac{-1215A^5-1}{2}-8505A^4B+48600A^3B^2+136080A^2B^3-194400AB^4-108864B^5 \quad (13)$$

$$y=\frac{-1539A^5-1}{2}+5265A^4B+61560A^3B^2-84240A^2B^3-246240AB^4+67392B^5 \quad (14)$$

$z = 9A^2 + 81B^2$ (15)

Choosing A to be odd in the above equations (13)-(15), one obtains the corresponding integer solutions to (1)

A few examples are presented below:

A	B	x	y	z
1	1	-127697	-197033	81
1	2	-5328626	-1954976	297
3	1	1008328	472108	153

Note that in (12), 33 may also be represented as given in (*) and corresponding one may obtain two more integer solutions for the given quintic equation.

3. Conclusion:

In this paper, we have presented two different patterns of non-zero distinct integer solutions to ternary quintic equation under consideration. As these equations are rich in variety one may attempt for obtaining integer solutions to other choices of quintic equations with variables greater than or equal to 3 and obtain their corresponding properties.

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