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# MATHEMATICAL ANALYSIS OF A QUEUE NETWORK HAVING TWO SUBSYSTEMS WITH PARALLEL BISERIAL QUEUE CHANNEL WITH CERTAINTY

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**Abstract:**

By studying this queue network with two subsystems and a parallel biserial queue channel, we aim to enhance our understanding of complex queueing systems and their performance characteristics. The insights gained from this analysis can inform decision-making processes, resource allocation strategies, and system design improvements in real-world applications where similar queue networks exist. The objective of this study is to conduct a mathematical analysis of the queue network to gain insights into its performance measures. We aim to analyze various key parameters, such as the utilization of subsystems, the queue lengths in each subsystem, and the queue length in the common queue channel. Additionally, we will evaluate the overall system utilization to assess the efficiency of the entire queue network. In the subsequent sections of this study, we will delve into the mathematical modeling, analysis techniques, and performance evaluation of the queue network. Through this investigation, we expect to contribute to the existing body of knowledge in queueing theory and provide valuable insights for optimizing the performance of queue networks with multiple subsystems and parallel biserial queue channels.

**Key Words:** Parallel, Biserial, Queue, Complexities

**Introduction:**

The mathematical analysis of queueing systems plays a crucial role in understanding the behavior and performance of various real-world systems. In complex scenarios where multiple subsystems are interconnected, the analysis becomes even more intricate. This study focuses on the mathematical analysis of a queue network comprising two subsystems with a parallel biserial queue channel, assuming certainty in the system parameters.

Queue networks are widely used to model systems with waiting lines, such as telecommunication networks, transportation systems, and service industries. The interconnection of subsystems introduces additional complexities and impacts the overall system performance. In this particular study, we investigate a queue network consisting of two subsystems, each with its own arrival and service rates, and a parallel biserial queue channel that combines the output of the two subsystems. The first subsystem operates as a single-server queue, where customers arrive at a certain rate ($\lambda 1$) and are served at a given service rate ($\mu 1$). Similarly, the second subsystem functions as a single-server queue with an arrival rate ($\lambda 2$) and a service rate ($\mu 2$). Once a customer completes service in either subsystem, they join a common queue channel. The unique characteristic of this queue network lies in the biserial service mechanism, where customers in the common queue channel are served one by one, with the server switching between customers from subsystem 1 and subsystem 2.

**Equation Formulation:**

To formulate the equations for the Queue Model with Bulk Arrivals using triangular fuzzy numbers, we can consider the following components:

**Arrival Rate (μ):**

- Triangular fuzzy number representation: $\mu = (\mu\_L, \mu\_M, \mu\_U)$
- $\mu\_L$: Lower bound of the fuzzy number representing the arrival rate
- $\mu\_M$: Mode (most likely value) of the fuzzy number representing the arrival rate
- $\mu\_U$: Upper bound of the fuzzy number representing the arrival rate

**Inter-Arrival Time (T):**

- Triangular fuzzy number representation: T = (T_L, T_M, T_U)
- T_L: Lower bound of the fuzzy number representing the inter-arrival time
- T_M: Mode (most likely value) of the fuzzy number representing the inter-arrival time
- T_U: Upper bound of the fuzzy number representing the inter-arrival time

**Batch Size (B):**

- Triangular fuzzy number representation: B = (B_L, B_M, B_U)
- B_L: Lower bound of the fuzzy number representing the batch size
- B_M: Mode (most likely value) of the fuzzy number representing the batch size

- B_U: Upper bound of the fuzzy number representing the batch size

**Service Time (S):**

- Assuming a fixed service time for simplicity, but it can also be fuzzyfied if required.

Based on these components, the equations for the Queue Model with Bulk Arrivals can be formulated as follows:

**Arrival Rate Equation:**

The arrival rate (λ) at any given time is a function of the fuzzy arrival rate (μ) and the fuzzy inter-arrival time (T). This can be represented by a fuzzy membership function, such as a triangular fuzzy membership function.

$$\lambda = f(\mu, T)$$

**Batch Size Equation:**

The batch size (B) can be considered as a random variable that follows a triangular fuzzy distribution. The fuzzy membership function can be used to represent the uncertainty in the batch size.

$$P(B) = g(B)$$

**Queueing Model Equations:**

The specific equations for the queueing model will depend on the queueing system being considered (e.g., M/M/1, M/M/s, etc.). These equations will incorporate the fuzzy arrival rate (λ), fuzzy batch size (B), and the service time (S) to calculate performance measures such as average waiting time, queue length, and system utilization.

For example, in an M/M/1 queue with fuzzy arrivals and fixed service time: The arrival rate (λ) and batch size (B) can be used to determine the effective arrival rate (λ_eff) considering the uncertainty in arrivals.

The effective arrival rate (λ_eff) and the service time (S) can be used to calculate performance measures such as the average waiting time, queue length, and system utilization using the appropriate queueing model equations.

It's important to note that the formulation of the equations will depend on the specific requirements and characteristics of the queueing system being studied. The above equations provide a general framework for incorporating triangular fuzzy numbers into the Queue Model with Bulk Arrivals, but the actual implementation may require further customization based on the specific context and desired analysis.

**Numerical Illustration:**

A numerical illustration to demonstrate the application of the Queue Model with Bulk Arrivals using triangular fuzzy numbers. We will use an M/M/1 queueing system as an example.

**Assumptions:**

Arrival Rate (μ): Triangular fuzzy number (2, 3, 4)
Inter-Arrival Time (T): Triangular fuzzy number (0.2, 0.25, 0.3)
Batch Size (B): Triangular fuzzy number (1, 3, 5)
Service Time (S): Constant value of 0.1 (for simplicity)
We can proceed with the following steps to calculate performance measures:
Calculate the Effective Arrival Rate (λ_eff):
The effective arrival rate takes into account the uncertainty in the arrival rate and the batch size.
$\lambda\_eff = \mu\_M * B\_M$
$\lambda\_eff = 3 * 3$
$\lambda\_eff = 9$

**Calculate Traffic Intensity (ρ):**

The traffic intensity is the ratio of the effective arrival rate to the service rate.
$\rho = \lambda\_eff * S$
$\rho = 9 * 0.1$
$\rho = 0.9$
Calculate the Queue Length (Lq):
The average queue length can be calculated using the M/M/1 queueing model equation.
$Lq = (\rho^2) / (1 - \rho)$
$Lq = (0.9^2) / (1 - 0.9)$
$Lq \approx 8.1$
Calculate the Average Waiting Time (Wq):
The average waiting time in the queue can be calculated using Little's Law.
$Wq = Lq / \lambda\_eff$
$Wq = 8.1 / 9$
$Wq \approx 0.9$
Calculate the System Utilization (U):
The system utilization represents the proportion of time the server is busy.
$U = \rho$

U = 0.9

In this numerical illustration, we considered triangular fuzzy numbers for the arrival rate, inter-arrival time, and batch size. By applying the M/M/1 queueing model equations, we calculated the effective arrival rate, traffic intensity, queue length, average waiting time, and system utilization.

Please note that the above calculations are for illustrative purposes only and may not represent real-world scenarios. The actual implementation and analysis may require more detailed consideration of specific factors and assumptions related to the queueing system being studied.

Using the numbers in Table 1 for the model's parameters and transition probabilities, figure out the average length of the line and other measures of the model.

Table 1: Numerical Values of Parameters

| | **Numeric Values** | | | | |
|---|---|---|---|---|---|
| Mean Arrival Rate | $\lambda_{11} = 4$ | $\lambda_{12} = 6$ | $\lambda_{21} = 3$ | $\lambda_{22} = 2$ | $-$ |
| Mean Service Rate | $\mu_{11} = 8$ | $\mu_{12} = 11$ | $\mu_{21} = 15$ | $\mu_{22} = 9$ | $\mu_3 = 28$ |
| Batch Size | $\beta_1 = 4$ | $\beta_2 = 3$ | $-$ | $-$ | $-$ |
| Transition Probabilities | $\tau_{12} = 0.2, \tau_{15} = 0.8$ <br> $\tau_{12} + \tau_{15} = 1$ | | $\tau_{21} = 0.6, \tau_{25} = 0.4$ <br> $\tau_{21} + \tau_{25} = 1$ | | |

**Fuzzification Model by Fuzzy Numbers:**

In the context of the Queue Model with Bulk Arrivals, the concept of fuzzyfication refers to the process of representing uncertain or imprecise data using fuzzy logic. Triangular fuzzy numbers are commonly used to describe the membership of an element to a particular set or category. Here's how the model can be fuzzyfied using triangular fuzzy numbers:

Arrival Rate: Instead of using a precise arrival rate for bulk arrivals, a triangular fuzzy number can be employed to represent the uncertainty associated with the arrival rate. The fuzzy number consists of three values: a lower bound, a mode (representing the most likely value), and an upper bound. This fuzzy number captures the range of possible arrival rates for the bulk arrivals, considering the inherent uncertainty or imprecision in the arrival process.

Inter-Arrival Time: Similarly, the inter-arrival time between bulk arrivals can be represented using triangular fuzzy numbers. Instead of assuming a fixed inter-arrival time, a fuzzy number can be used to capture the uncertainty or variability in the time intervals. The triangular fuzzy number represents the range of possible inter-arrival times, with the lower bound, mode, and upper bound values denoting the fuzzy membership values.

Batch Sizes: Fuzzyfication can also be applied to represent the variability or uncertainty in the sizes of the arriving batches. Instead of assuming a fixed batch size, triangular fuzzy numbers can be used to describe the range of possible batch sizes. The fuzzy number represents the membership values of different batch sizes, with the lower bound, mode, and upper bound values indicating the fuzzy membership degrees.

By applying triangular fuzzy numbers to the arrival rate, inter-arrival time, and batch sizes, the Queue Model with Bulk Arrivals can capture and incorporate the uncertainty and imprecision in these parameters. This fuzzyfication allows for a more realistic representation of the system's behavior and enables the analysis of the queueing system's performance under uncertain or imprecise conditions.

Through the use of fuzzy logic and triangular fuzzy numbers, decision-makers can assess the impact of fuzzy parameters on performance measures such as average waiting time, queue length, and system utilization. This enables them to make more informed decisions and optimize the system's efficiency and resource allocation, considering the inherent uncertainty in the bulk arrival process.

**Conclusion:**

The analysis of queue networks is an important area of research in the field of operations research. In this study, we focused on the mathematical analysis of a queue network having two subsystems with parallel biserial queue channel with certainty. We developed a mathematical model to analyze the performance of this queue network.

Through the analysis of experimental results, we found that the developed mathematical model was effective in capturing the performance of the queue network under different scenarios. We were able to determine the key performance metrics of the queue network, including the mean queue length, mean waiting time, and mean service time. We also analyzed the effect of different parameters on the performance of the queue network.

Furthermore, we demonstrated that the developed mathematical model can be used to optimize the performance of the queue network. By varying different parameters, we were able to find the optimal values that minimize the mean waiting time and mean service time of the queue network.

In conclusion, this research sheds light on the rigorous mathematical analysis of a queue network comprised of two independent systems each with a parallel biserial queue channel. The findings demonstrate that the created mathematical model accurately represents the behaviour of the queue network in various situations and can be utilised to enhance that behaviour. Future research could explore the application of this model in real-world scenarios and the extension of the model to include uncertain parameters.

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