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GENERALIZATION OF L-FUZZY COUNTABLE COMPACT SPACES

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Abstract:

By means of γ-open L-sets and their inequality, countable γ-compactness and γ-Lindelof property are introduced in L- topological spaces, where L is a complete De Morgan algebra. They do not depend on the structure of basis lattice L and L does not require any distributivety. An L-set is γ-compact if and only if it is countably γ- compact and has the γ- Lindelof property.

1. Introduction:

It is known that compactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [2], various kinds of fuzzy compactness [2, 5, 11] have been established. However, these concepts of fuzzy compactness rely on the structure of L and L is required to be completely distributive. In [10], for a complete De Morgan algebra L, Shi introduced a new definition of fuzzy compactness in L-topological spaces using open L-sets and their inequality. This new definition does not depend on the structure of L. In this paper, by means of γ-open L-sets and their inequality, countable γ-compactness and γ--Lindelof property are introduced in L-topological spaces, wehre L is a complete De Morgan algebra. They do not depend on the structure of basis lattice L and L does not require any distributivity. An L-set is γ-compact if and only if it is countably γ-compact and has the γ- Lindelof property.

2. Preliminaries:

Throughout this proper, $(L,\vee,\wedge')$ is a complete De Morgan algebra, X a nonempty set and L^x the set of all L –fuzzy sets (or L-sets for short) on X. The smallest element and the largest element in L^x are denoted by $\underline{0}$ and $\underline{1}$. An element a is L is called a prime element if $b\wedge c\leq a$ implies that $b\leq a$ or $c\leq a$. a in L is called a co-prime element if a' is a prime element [4]. The set of nonunit prime elements in L is denoted by P(L) and the set of nonzero co-prime elements in L by M(L). The binary relation $\propto$ in L is defined as follows; for $a,b\in L, a\propto b$ if and only if for every subset $D\subseteq L$, the relation $b\leq \sup D$ always implies the existence of $d\in D$ with $a\leq d$ [3]. In a completely distributive De Morgan algebra L, each element b is a supof $\{a\in L\backslash\propto b\}$. The set $\beta(b)=\{ a\in L\backslash\propto b \}$ is called the greatest minimal family of b in the sense of [5,11]. Now, for $b\in L$, we define $\beta^*(b)=\beta(b)\cap M(L),\alpha(b)=\{A\in L\backslash a'\propto b'\}$ and $\alpha^*(b)= \alpha(b) \cap P(L)$. For $a\in L$ and $A\in L^x$, we write $A^{(\alpha)}=\{x\in X\backslash A(x) \leq\alpha\}$ and $A_{(\alpha)}=\{x\in X\backslash\alpha\in\beta(A(x))\}$ and for a subfamily $\psi\subseteq L^x$, $2(\psi)$, (resp.$2^{[\psi]}$) will denote the set of all finite (countable) subfamilies of ψ.A L-topological space is an ordered pair (X,τ), where τ is a subfamily of L^x which contains $\underline{0},\underline{1}$ and is closed for any suprema and finite infima. An L-topological space is an ordered triple (X, τ), where τ is a subfamily of L^x which contains $\underline{0},\underline{1}$, and is closed for3 any suprema and finite infima.

Definition 2.1 [5, 11]: For a topolotical space (X, τ) let $\omega_L(\tau)$ denote the family of all the lower semi continuous maps from (X, τ) to L; that is, $\omega_L(\tau)=\{A\in L^x|A^{(\alpha)}\in\tau, \alpha L\}$. Then $\omega_L(\tau)$ is an L-topology on X and $(X, \omega_L(\tau)))$ is topologically generated by (X, τ).

Definition 2.2 [5, 11]: An L-topological space (X, τ) is weak induced if, for all $\alpha\in L$ and for all $A\in\tau$, it follows that $A^{(\alpha)}\in[\tau]$, where [τ]denotes the topology formed by all crisp sets in τ. It is obvious that $(X, \omega_L(\tau))$ is weak induced.

Definition 2.3 [9]: Let (X, τ) be an L-topological space, $\alpha\in L\backslash\{1\}$, and $A\in L^x$. A family $\mu\subseteq L^x$ is called

1. an α-shading of A if for any $x\in X,(A'(x)\vee \bigvee_{\beta\in\mu} B(x))\not\leq\alpha$.
2. a strong a –shadining of A if $\bigwedge_{\beta\in\mu} A'(x)\vee \bigvee_{\beta\in\mu} B(x))\not\leq\alpha$.
3. an a R-neighborhood family of A if for any $x\in X$, $(A(x)\wedge \bigwedge_{\beta\in\mu} B(x))\not\geq\alpha$.
4. a strong a–R neighbourhood family of A if $\bigvee_{x\in X} (A(x)\wedge \bigwedge_{\beta\in\mu} B(x))\not\geq\alpha$.

It is obvious that strong a –shading of A is an a –shading of A, an strong a-R-neighbourhood family of A if an a-R-neighbourhood family of A if and only if μ'is an strong a –shading of A.

Definition 2.4 [9]: Let (X, τ) be an L-topological space, $\alpha\in L\backslash\{0\}$ and $A\in L^x$. A family $\mu\subseteq L^x$ is called

1. $\alpha\beta_\alpha$-cover of A if for any x∈X, it follows that $\alpha\in\beta(A'(x) \vee \bigvee_{B\in\mu} B(x))$.
2. a strong β_α -cover of A if $\alpha\in\beta(\bigwedge_{x\in X}(A'(x)) \vee \bigvee_{B\in\mu} B(x)))$.
3. α Q_α- cover of A if for any x∈X, it follows that $A'(x) \vee \bigvee_{B\in\mu} B(x)\geq \alpha)$.

It is obvious that an S-β_α-cover of A be a β_α-cover of A, and ${}_\alpha\beta_\alpha$-cover of A must be Q_α -cover of A.

Definition 2.5 [9]: Let $A\in L\backslash\{0\}$ and $A\in L^x$. A family $\mu\subset L^x$ is said to have weak a –nonempty intersection in a if $\bigvee_{x\in X} A(x)\wedge \beta(x))\geq \alpha$.μ is said to have the finite weak α- intersection property in A if every finite subfamily υof μ has weak α-nonempty intersection in A.

Lemma 2.6: Let L be a complete Heyting algebra, f:X→ Y a map and $f\vec{L}:L^X\to L^Y$ the extension of f. Then for any family $\psi\subseteq L^Y$, $\bigvee_{y\in Y}(f\vec{L}(A)(y)\wedge \bigwedge_{B\in\varphi} B(y)= \bigvee_{x\in X}(A(x)\wedge \bigwedge_{B\in\varphi}(B)(x))$.

Definition 2.7 [1]: Let (X, τ) be an L-topological space, $a\in L^x$. Then A is called a- γ open set if A≤ Int(Cl(A))VCl (Int(A)).The complement of an γ -open set is called an γ –closed set. Also, $\gamma O(L^x)$ and γ C(Lx) will always denote the family of all γ- open sets and γ –closed sets respectively. Obviously, $A\in O(L^x)$ if and only if $A'\in\gamma C(L^x)$.

Definition 2.8 [1]: Let (L^x, τ) be an L-topolotical space, $A,B\in L^X$. Let $\gamma Int(A)=V\{ B\in L^X \backslash B\leq A, B\in\gamma C(L^x)\}$, $\gamma Cl(A)=\wedge B\in L^X\backslash A\leq B, B\in\gamma C(L^x)\}$. Then γInt (A)and γCl(A) are called the γ –interior and γ-closure of A, respectively.

Definition 2.9 [6]: Let (X, τ) and (Y,σ) be two L-topological spaces. A map f:(X, τ)→ (Y, σ) is called

1. γ- continuous if $f\vec{L}$ (B)isγ-open in (X, τ) for every B∈σ.
2. γ- irresolute if $f\vec{L}$ (B)isγ-open in (X, τ) for every γ-open L-set B in (Y,σ).

Definition 2.10 [7]: Let (X, τ)be an L-topological space. $A\in L^X$ is called γ-compact if for every family μ of γ - open L-sets, it follows that

$\bigwedge_{x\in X}(A'(x)) \vee \bigvee_{\beta\in\mu} B(x)\leq \bigvee_{\nu\in 2(\mu)} \bigwedge_{x\in X}(A'(x) \vee \bigvee_{\beta\in\nu} B(x))$. (X, τ) is called γ – compact if $\underline{1}$ is γ – compact.

3. Properties of Countable γ – Compactness:

Definition 3.1: Let (X, τ)be an L-topological space. $A\in L^X$ is called countable γ-compact if for every family μ of γ - open L-sets, it follows that $\bigwedge_{x\in X}(A'(x)) \vee \bigvee_{\beta\in\mu} B(x)\leq \bigvee_{\nu\in 2(\mu)} \bigwedge_{x\in X}(A'(x) \vee \bigvee_{\beta\in\nu} B(x))$. Then (X, τ) is called countable γ-compact if $\underline{1}$ is countable γ – compact.

Definition 3.2: Let(X, τ) be an L-topological space. $A\in L^X$ is said to have the γ-Lindelof property if for every family μ of γ - open L-sets, it follows that $\bigwedge_{x\in X}(A'(x)) \vee \bigvee_{\beta\in\mu} B(x)\leq \bigvee_{\nu\in 2(\mu)} \bigwedge_{x\in X}(A'(x) \vee \bigvee_{\beta\in\nu} B(x))$. Then (X, τ) have the γ – Lindelof property if $\underline{1}$ have the γ – Lindelof property. Obviously, we have the following theorem.

Theorem 3.3: Let (X, τ) be an L-topological space. $A\in L^X$ γ-compact if and only if A is countably γ-compact and has the γ-Lindelof property.

Theorem 3.4: Let (X, τ) be an L-topolotical space. $A\in L^X$ is countably γ-compact if and only if for every family μ of γ - closed L-sets, it follows that $\bigvee_{x\in X}(A'(x))\wedge \bigwedge_{\beta\in\mu} B(x)\geq \bigwedge_{\nu\in 2(\mu)} \bigvee_{x\in X}(A'(x \wedge \bigwedge_{B\in\nu} B(x))$.

Proof: This is immediate from Definition 3.1 and quasi complements.

Theorem 3.5: Let (X, τ) be an L-topolotical space. $A\in L^X$ has the γ-Lindelof property if and only if for every family μ of γ –closed L-sets, it follows that

$$\bigvee_{x\in X}(A'(x))\wedge \bigwedge_{\beta\in\mu} B(x)\geq \bigwedge_{\nu\in 2(\mu)} \bigvee_{x\in X}(A'(x \wedge \bigwedge_{B\in\nu} B(x)).$$

Proof: This is immediate from Definition 3.2 and quasi complements.

Theorem 3.6: Let (X, τ) be an L-topological space and $A\in L^X$. Then the following conditions are equivalent.

1. A is countably γ - compact
2. For any $a\in L\backslash\{1\}$, each countably γ – open strong α- shading μ of A has a finite subfamily which is an strong a –shading of A.
3. For any $a\in L\backslash\{0\}$' each countably γ– closed strong α- R- q neighbourhood family ψof A has a finite subfamily which is an strong α- R- neighbourhood family of A.

4. For any $a\in L\backslash\{0\}$' each countably family of γ– closed L-sets which has the finite weak α- intersection property in A has weakly α-nonempty intersection in A.

Theorem 3.7: Let(X, τ) be an L-topological space and $A\in L^X$. Then the following conditions are equivalent.

1. A has the γ – Lindelof property
2. For any $a\in L\backslash\{1\}$, each γ – open strong α- shading μ of A has a countably subfamily which is an strong a –shading of A.
3. For any $a\in L\backslash\{0\}$' each countable γ– closed strong α- R- q neighbourhood family $\psi$of A has a finite subfamily which is an strong α- R- neighbourhood family of A.
4. For any $a\in L\backslash\{0\}$' each countable family of γ– closed L-sets which has the finite weak α- intersection property in A has weakly α-nonempty intersection in A.

Theorem 3.8: Let L be a complete Heyting algebra. If both C and D are countably γ – compact (γ -Lindelof), then CvD is countable γ – compact (γ -Lindelof).

Proof: We only prove that the theorem is true for countable γ –compactness. By Theorem 3.5, for any family μ of γ –closed L-sets, we have

$$\bigvee_{x\in X}((C\vee D)(x)\wedge\bigwedge_{B\in\mu}B(x))$$

$$=\{\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in\mu}B(x))\}\vee\{\bigvee_{x\in X}(D(x)\wedge\bigwedge_{B\in\mu}B(x))\}$$

$$\geq\{\bigwedge_{v\in 2(\mu)}\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in\mu}B(x))\}\vee\{\bigwedge_{v\in 2(\mu)}\bigvee_{x\in X}(D(x)\wedge\bigwedge_{B\in\mu}B(x))\}$$

$$=\bigwedge_{v\in 2(\mu)}\bigvee_{x\in X}(C\vee D)(x)\wedge\bigwedge_{B\in v}B(x)).$$

This shows that CvD is countably γ – compact.

Theorem 3.9: Let (X, τ) be an L-topological space and C, $D\in L^X$. If C is countably γ –compact (γ –Lindelof) and D is γ –closed, then $C\wedge D$ is countably γ –compact (γ -Lindelof).

Proof: We only prove that the theorem is true for countable γ –compactness. By Theorem 3.5, for any family μ of γ –closed L-sets, we have

$$\bigvee_{x\in X}((C\wedge D)(x)\wedge\bigwedge_{B\in v}B(x)).$$

$$=\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in\mu U(D)}B(x)).$$

$$\geq\bigwedge_{v\in 2(\mu U(D))}\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in v}B(x)).$$

$$\geq\bigwedge_{v\in 2(\mu)}\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in v}B(x))\}\wedge\bigwedge_{v\in 2(\mu U(D))}\bigvee_{x\in X}(C(x)\wedge D(x)\wedge\bigwedge_{B\in v}B(x))\}$$

$$=\bigwedge_{v\in 2(\mu)}\bigvee_{x\in X}((C\wedge D)(x)\wedge\bigwedge_{B\in v}B(x))$$

This shows that $C\wedge D$ is countably γ –compact.

Theorem 3.10: Let L be a complete Heyting algebra and f:(X, τ)$\rightarrow$(Y,σ) be a γ – irresolute map. If A is a countably γ –compact (γ –Lindelof) L-set in (X, τ), then $f\vec{L}$ (A) is an countably γ –compact (γ -Lindelof) L- set in (Y,σ).

Proof: We only prove that the theorem is true for countably γ –compact. Suppose that μ is a countable family of γ –closed L-sets in (Y,σ), we have that $\bigvee_{y\in Y} f\vec{L}(A)(y)\wedge\bigwedge_{B\in\mu}B(y))=\bigvee_{x\in X}((A)(x)\wedge\bigwedge_{B\in\mu}f\vec{L}(B)(x))\geq$

$$=\bigwedge_{v\in 2(\mu)}\bigvee_{x\in X}((A(x)\wedge\bigwedge_{B\in v}f\vec{L}B(x))=\bigwedge_{v\in 2(\mu)}\bigvee_{y\in Y}f\vec{L}(A)(y)\wedge\bigwedge_{B\in v}B(y)).$$

Hence $f\vec{L}$ (A) is countably γ - compact.

Analogously, we have the following result.

Theorem 3.11: Let L be a complete Heyting algebra and f:(X, τ)$\rightarrow$(Y, τ) be an γ – continuous map. If A is a countably γ –compact (γ –Lindelof) L-set in (X, τ), then $f\vec{L}$ (A) is an countably γ –compact (Lindelof) L- set in (Y, τ).

Now, we assume that L is a completely distributive de Morgan algebra.

Theorem 3.12: Let (X,τ) be an L-topological space and A εL^X . Then the following conditions are equivalent.

1. A is countably γ compact.
2. For any $\alpha\varepsilon L\setminus\{0\}$, each countably γ closed strong α -R-neighbourhood family ψ of A has a finite subfamily which is an strong α -R-neighbourhood family of A.
3. For any $\alpha\varepsilon L\setminus\{0\}$, each countably γ closed strong α -R-neighbourhood family ψ of A has a finite subfamily which is an α -R-neighbourhood family of A.
4. For any $\alpha\varepsilon L\setminus\{0\}$, each countably γ closed strong α -R-neighbourhood family ψ of A, there exist a finite subfamily ϕ of ψ and b $\varepsilon\beta(\alpha)$ such that ϕ is an S-b-R-neighbourhood family of A.
5. For any $\alpha\varepsilon L\setminus\{0\}$, each countably γ closed strong α -R-neighbourhood family ψ of A, there exist a finite subfamily ϕ of ψ and b $\varepsilon\beta(\alpha)$ such that ϕ is a b-R-neighbourhood family of A.
6. For any $\alpha\varepsilon M(L)$ each countably γ closed strong α -R-neighbourhood family ψ of A has a finite subfamily which is an strong $\alpha -$ R-neighbourhood family of A.
7. For any $\alpha\varepsilon M(L)$ each countably γ closed strong α -R-neighbourhood family ψ of A has a finite subfamily which is an $\alpha -$ R-neighbourhood family of A.
8. For any $\alpha\varepsilon M(L)$ each countably γ closed strong α -R-neighbourhood family ψ of A, there exist a finite subfamily ϕ of ψ and b $\varepsilon\beta^*(\alpha)$ such that ϕ is an S-b-R-neighbourhood family of A.
9. For any $\alpha\varepsilon M(L)$ each countably γ closed strong α -R-neighbourhood family ψ of A, there exist a finite subfamily ϕ of ψ and b $\varepsilon\beta^*(\alpha)$ such that ϕ is an α b-R-neighbourhood family of A.
10. For any $\alpha\varepsilon L\setminus\{1\}$, each countably γ open strong α shading μ of A has a finite subfamily which is an strong α - shading of A.
11. For any $\alpha\varepsilon L\setminus\{1\}$, each countably γ open strong α shading μ of A has a finite subfamily which is an α - shading of A.
12. For any $\alpha\varepsilon L\setminus\{1\}$, and any countably γ open strong α shading μ of A, there exist a finite subfamily ν is an strong S-b shading of A.
13. For any $\alpha\varepsilon L\setminus\{1\}$, and any countably γ open strong α shading μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\,\alpha\,(\alpha)$ such that ν is α b shading of A.
14. For any $\alpha\,\varepsilon\,P(L)$, each countably γ open strong α shadining μ of A has a finite subfamily which is an strong α shading of A.
15. For any $\alpha\,\varepsilon\,P(L)$, each countably γ open strong α shadining μ of A has a finite subfamily which is an strong α shading of A.
16. For any $\alpha\,\varepsilon\,P(L)$, and any countably γ open strong α - shading μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\,\alpha^*(\alpha)$ such that ν is an S-b- shadining of A.
17. For any $\alpha\,\varepsilon\,P(L)$, and any countably γ open strong α - shading μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\,\alpha^*(\alpha)$ such that ν is a S-b- shadining of A.
18. For any $\alpha\,\varepsilon\,L\setminus\{0\}$ and any countably γ open α - shading μ open S-$\beta_\alpha -$ cover μ of A has a finite subfamily which is an S-$\beta_\alpha -$ cover of A.
19. For any $\alpha\,\varepsilon\,L\setminus\{0\}$ and any countably γ open S-$\beta_\alpha -$ cover μ of A has a finite subfamily which is $\alpha\ \beta_\alpha -$ cover of A.
20. For any $\alpha\,\varepsilon\,L\setminus\{0\}$ and any countably γ open S-$\beta_\alpha -$ cover μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\,L$ with $\alpha\ \varepsilon\ \beta(b)-$ such that ν is an S-β_α cover of A.
21. For any $\alpha\,\varepsilon\,L\setminus\{0\}$ and any countably γ open S-$\beta_\alpha -$ cover μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\,L$ with $\alpha\ \varepsilon\ \beta(b)-$ such that ν is α S-β_α cover of A.

22. For any $\alpha \,\varepsilon\, M\,(L)$, each countably γ open S- β_α cover μ of A has a finite subfamily which is an S- β_α cover of A.
23. For any $\alpha \,\varepsilon\, M\,(L)$, each countably γ open S- β_α cover μ of A has a finite subfamily which is α β_α cover of A.
24. For any $\alpha \,\varepsilon\, M(L)$ and any countably γ open S- $\beta_\alpha -$ cover μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\, M(L)$ with $\alpha \;\varepsilon\; \beta^*(b) -$ such that ν is an S- β_α cover of A.
25. For any $\alpha \,\varepsilon\, M(L)$ and any countably γ open S- $\beta_\alpha -$ cover μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\, M(L)$ with $\alpha \;\varepsilon\; \beta^*(b)$ such that ν is α β_α cover of A.
26. For any $\alpha \,\varepsilon\, L\setminus\{0\}$ and any b $\varepsilon\, \beta(\alpha)\setminus\{0\}$, each countably γ open $Q_\alpha -$ cover μ of A has a finite subfamily which is α Q_b cover of A.
27. For any $\alpha \,\varepsilon\, L\setminus\{0\}$ and any b $\varepsilon\, \beta(\alpha)\setminus\{0\}$, each countably γ - open $Q_\alpha -$ cover μ of A has a finite subfamily which is α Q_b cover of A.
28. For any $\alpha \,\varepsilon\, L\setminus\{0\}$ and any b $\varepsilon\, \beta(\alpha)\setminus\{0\}$, each countably γ - open $Q_\alpha -$ cover μ of A has a finite subfamily which is α S- β_b cover of A.
29. For any $A \,\varepsilon\, M\,(L)$, and any b $\varepsilon\, \beta^*(\alpha)$ each countably γ open Q_α cover μ of A has a finite subfamily which is a Q_b cover of A.
30. For any $A \,\varepsilon\, M\,(L)$ and any b $\varepsilon\, \beta^*(\alpha)$ each countably γ open Q_α cover μ of A has a finite subfamily which is a Q_b cover of A.
31. For any $A \,\varepsilon\, M\,(L)$ and any b $\varepsilon\, \beta^*(\alpha)$ each countably γ open Q_α cover μ of A has a finite subfamily which is a S- β_b cover of A.

Proof: (1) $\Rightarrow$ (2) : This follows directly from Theorem 3.7
(2) $\Rightarrow$ (3) : This is easy to prove if one notices that every strong $\alpha - R -$ neighbourhood family of A is an $\alpha - R -$ neighbourhood family of A.
(3) $\Rightarrow$ (4): Let $\alpha \,\varepsilon\, L\setminus\{0\}$ and ψ is a countably γ closed strong $\alpha - R -$ neighbourhood family of A. Then $\underset{x\varepsilon X}{V}(A(x) \wedge \underset{B\varepsilon\psi}{\wedge} B(x)) \not\geq \alpha$. Take c ε β(α) such that $\underset{x\varepsilon X}{V}(A(x) \wedge \underset{B\varepsilon\psi}{\wedge} B(x)) \not\geq$ c. Obviously ψ is an γ closed S-c-R- neighbourhood family of A. By (3) we know that ψ has a finite subfamily ϕ which is a c-R-neighbourhood family of A. Take b εβ(α) such that c εβ(b), then ϕ is an S-b-R- neighbourhood family of A.
(4) $\Rightarrow$ (5) $\Rightarrow$ (2): Obvious.
(2) $\Rightarrow$) $\Rightarrow$ (7) $\Rightarrow$ (8) $\Rightarrow$ (9) $\Rightarrow$ (1) : the proof is similar.
(1) $\Leftrightarrow$ (10) : This follows directly from Theorem 3.7
(10) $\Leftrightarrow$ (11) : This is easy to prove if one notices that every strong α - shading of A is an α-shading of A.
(11) $\Leftrightarrow$ (12) : Let αε L\{1} and μ be a countably γ open strong α - shading of A. Then

$$\underset{x\varepsilon X}{\wedge}(A'(x) \vee \underset{B\varepsilon\mu}{V}\; B(x)) \not\leq \alpha. \text{ Take } c \in \alpha(a)$$

such that $\underset{x\varepsilon X}{\wedge}(A'(x) \vee \underset{B\varepsilon\mu}{V}\; B(x)) \not\leq$ c. Obviously μ is an γ -open S-c- shadining of A. By (11) we know that μ has a finite subfamilyvwhich is a c-shadnining of A. Take b∈ α(a) such that c∈ α(b), then v is an S-b-shading of A.
(12) $\Rightarrow$ (13) $\Rightarrow$ (10): Obvious
(10) $\Rightarrow$ (14) $\Rightarrow$ (15) $\Rightarrow$ (16) $\Rightarrow$ (17) $\Rightarrow$ (10): We can prove these in the similar way. Similarly we can prove the other results.

Theorem 3.13: Let (X,τ) be an L-topological space and A∈L^X. Then the following conditions are equivalent.

1. A has the γ -Lindelof property.

2. For any $\alpha \varepsilon L \setminus \{0\}$, each γ - closed strong α -R-neighbourhood family ψ of A has a countable subfamily which is an strong α -R-neighbourhood family of A.
3. For any $\alpha \varepsilon L \setminus \{0\}$, each γ -closed strong α -R-neighbourhood family ψ of A has a countable finite subfamily which is an α -R-neighbourhood family of A.
4. For any $\alpha \varepsilon L \setminus \{0\}$, each countably γ closed strong α -R-neighbourhood family ψ of A, there exist a finite subfamily ϕ of ψ and b $\varepsilon \beta(\alpha)$ such that ϕ is an S-b-R-neighbourhood family of A.
5. For any $\alpha \varepsilon L \setminus \{0\}$, each γ - closed strong α -R-neighbourhood family ψ of A, there exist a countable subfamily ϕ of ψ and b $\varepsilon \beta(\alpha)$ such that ϕ is a b-R-neighbourhood family of A.
6. For any $\alpha \varepsilon M(L)$ each γ -closed strong α -R-neighbourhood family ψ of A has a countable subfamily which is an strong $\alpha -$ R-neighbourhood family of A.
7. For any $\alpha \varepsilon M(L)$ and any γ - closed strong α -R-neighbourhood family ψ of A has a countable subfamily which is an $\alpha -$ R-neighbourhood family of A.
8. For any $\alpha \varepsilon M(L)$ and any γ - closed strong α -R-neighbourhood family ψ of A, there exist a countable subfamily ϕ of ψ and b $\varepsilon \beta^*(\alpha)$ such that ϕ is an S-b-R-neighbourhood family of A.
9. For any $\alpha \varepsilon M(L)$ and any γ - closed strong α -R-neighbourhood family ψ of A, there exist a countable subfamily ϕ of ψ and b $\varepsilon \beta^*(\alpha)$ such that ϕ is an α b-R-neighbourhood family of A.
10. For any $\alpha \varepsilon L \setminus \{1\}$, each - γ -open strong α shading μ of A has a countable subfamily which is an strong α - shading of A.
11. For any $\alpha \varepsilon L \setminus \{1\}$, each γ - open strong α - shading μ of A has a finite subfamily which is an α - shading of A.
12. For any $\alpha \varepsilon L \setminus \{1\}$, and any γ - open strong α shading μ of A, there exist a finite subfamily ν of μ and $b \varepsilon \alpha(a)$ such that v is an S-b shadining of A.
13. For any $\alpha \varepsilon L \setminus \{1\}$, and any γ -open strong α shading μ of A, there exist a finite subfamily ν of μ and b $\varepsilon\, \alpha\, (\alpha)$ such that ν is α b shading of A.
14. For any $\alpha\, \varepsilon\, P(L)$, each γ open strong α shadining μ of A has a countable subfamily which is an strong α shading of A.
15. For any $\alpha\, \varepsilon\, P(L)$, each γ - open strong α shadining μ of A has a countable subfamily which is an strong α shading of A.
16. For any $\alpha\, \varepsilon\, P(L)$, and any γ open strong α - shading μ of A, there exist a countable subfamily ν of μ and b $\varepsilon\, \alpha^*(\alpha)$ such that ν is an S-b- shadining of A.
17. For any $\alpha\, \varepsilon\, P(L)$, and any - γ open strong α - shading μ of A, there exist a countable subfamily ν of μ and b $\varepsilon\, \alpha^*(\alpha)$ such that ν is a S-b- shadining of A.
18. For any $\alpha\, \varepsilon\, L \setminus \{0\}$ and each γ - open α - shading μ open S- $\beta_\alpha -$ cover μ of A has a countable subfamily which is an S- $\beta_\alpha -$ cover of A.
19. For any $\alpha\, \varepsilon\, L \setminus \{0\}$ and each γ - open S- $\beta_\alpha -$ cover μ of A has a countable subfamily which is $\alpha\, \beta_\alpha -$ cover of A.
20. For any $\alpha\, \varepsilon\, L \setminus \{0\}$ and and γ open S- $\beta_\alpha -$ cover μ of A, there exist a countable subfamily ν of μ and b $\varepsilon\, L$ with $\alpha\, \varepsilon\, \beta(b) -$ such that ν is an S- β_α cover of A.
21. For any $\alpha\, \varepsilon\, L \setminus \{0\}$ and any countably γ open S- $\beta_\alpha -$ cover μ of A, there exist a countable subfamily ν of μ and b $\varepsilon\, L$ with $\alpha\, \varepsilon\, \beta(b) -$ such that ν is α S- β_α cover of A.
22. For any $\alpha\, \varepsilon\, M\, (L)$, each γ open S- β_α cover μ of A has a countable subfamily which is an S- β_α cover of A.
23. For any $\alpha\, \varepsilon\, M\, (L)$, each γ open S- β_α cover μ of A has a subfamily which is $\alpha\, \beta_\alpha$ cover of A.

24. For any $\alpha \varepsilon M(L)$ and any γ - open S- $\beta_\alpha -$ cover μ of A, there exist a countable subfamily ν of μ and b $\varepsilon M(L)$ with $\alpha \varepsilon \beta^*(b)-$ such that ν is an S- β_α cover of A.
25. For any $\alpha \varepsilon M(L)$ and any γ - open S- $\beta_\alpha -$ cover μ of A, there exist a countable subfamily ν of μ and b $\varepsilon M(L)$ with $\alpha \varepsilon \beta^*(b)$ such that ν is α β_α cover of A.
26. For any $\alpha \varepsilon L\backslash\{0\}$ and any b $\varepsilon \beta(\alpha)\backslash\{0\}$, each γ - open $Q_\alpha -$ cover μ of A has a countable subfamily which is α Q_b cover of A.
27. For any $\alpha \varepsilon L\backslash\{0\}$ and any b $\varepsilon \beta(\alpha)\backslash\{0\}$, each γ - open $Q_\alpha -$ cover μ of A has a countable subfamily which is α Q_b cover of A.
28. For any $\alpha \varepsilon L\backslash\{0\}$ and any b $\varepsilon \beta(\alpha)\backslash\{0\}$, each γ - open $Q_\alpha -$ cover μ of A has a countable subfamily which is α S- β_b cover of A.
29. For any $A \varepsilon M(L)$, and any b $\varepsilon \beta^*(\alpha)$ each γ - open Q_α cover μ of A has a countable subfamily which is a Q_b cover of A.
30. For any $A \varepsilon M(L)$ and any b $\varepsilon \beta^*(\alpha)$ each γ - open Q_α cover μ of A has a countable subfamily which is a Q_b cover of A.
31. For any $A \varepsilon M(L)$ and any b $\varepsilon \beta^*(\alpha)$ each countably γ open Q_α cover μ of A has a finite subfamily which is a S- β_b cover of A.

Lemma 3.14: Let (X, ω_L(T)) be generated by (X, τ). If A is an γ – open L-set in (X, τ), then X_A is an γ – open set in (X, ω_L(T)). If B is an γ – open L-set in (X, ω_L(T)), then $B_{(a)}$ is an γ – open set in (X, τ) for every a∈L.
Proof. This is easy to prove if one notices that $X_{D^-}=(X_D)^-$ and $X_{Do}=(X_D)$- and $X_{Do}=(X_D)_0$ and applies Lemma 5.4 in [9].

Theorem 3.15: Let (X,τ) be a topological space and(X, ω_L(T)) be generated by (X,τ). Then (X, ω_L(T)) is countably γ – compact (γ - Lindelof) if and only if (X,τ) is countably γ – compact (γ – Lindelof).

Proof: We only prove that the theorem is true for countabl γ – compactness. Letμ be a countable γ –open cover of (X,τ).

Then (X_A|A∈μ} is a family of γ–open L-sets in (X, ω_L(T)) with $\bigwedge_{x\varepsilon X}(\bigvee_{A\varepsilon\mu} X_A(x)=1$. From countable γ – compactness of (X, ω_L(T)). we have that 1= $\bigwedge_{x\varepsilon X}(\bigvee_{A\varepsilon\mu} X_A(x) \le \bigvee_{\nu\in 2\mu}\bigwedge_{A\in\nu} X_A(x)$. This implies that there exists υ∈$2^{(\mu)}$ such that $\bigwedge_{x\varepsilon X}(\bigvee_{A\varepsilon\mu} X_A(x)=1$. Hence υ is a cover of (X,τ). Therefore (X,τ) is countable γ – compact. Conversely, let μbe a countable family of γ- open L-sets in (X, ω_L(T)) and $\bigwedge_{x\varepsilon X}(\bigvee_{B\varepsilon\mu} B(x))$ =a. If a=0, obviously we have that $\bigwedge_{x\varepsilon X}(\bigvee_{B\varepsilon\mu} B(x)) \le \bigvee_{\nu\in 2\mu}\bigwedge_{x\in X}(\bigvee_{B\in\nu}$ B(x)). Now we suppose that a ≠0. In this case, for any b∈β(a)

By lemma 3.14, this implies that {B(b)|B∈μ} is a countableγ- open cover of (X,ɩ) and from the countable γ-compactness of (X,ɩ) we know that there exists v∈$2^{(\mu)}$ such that {B(b)|B∈v } is a cover of (X,ɩ).

Hence b≤ $\bigwedge_{x\in X}(\bigvee_{B\in\nu}$ B(x)).

Moreover, we have that b≤ $\bigwedge_{x\in X}(\bigvee_{B\in\nu}$ B(x)) ≤ $\bigvee_{\nu\in 2((\mu)}\bigwedge_{x\in X}(\bigvee_{B\in\nu}$ B(x)).

This implies that $\bigwedge_{x\in X}(\bigvee_{B\in\nu}$ B(x))=a=v{b|b∈B(a))≤ $\bigvee_{\nu\in 2((\mu)}\bigwedge_{x\in X}(\bigvee_{B\in\nu}$ B(x)).

Therefore (X,$w_L^{(T)}$) is countable γ-compact.

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