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A HISTORICAL OVERVIEW OF VIKOR MODEL (VISE KRITERIJUMSKA OPTIMIZACIJA I KOMPROMISNO RESENJE)

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Abstract:

This paper enumerates the development and modifications of VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje) Model in different fields. We primarily focus on the growth and advancements of VIKOR model throughout the past four decades starting from 1973.VIKOR model has been frequently applied in multi criteria decision making problems. Further, we discuss the various recent progressions of VIKOR especially with reference to fuzzy set theory.

Key Words: VIKOR, Fuzzy Set, MCDM & Compromise Solution

1. Introduction:

Decision making is an important daily task widely experienced by humans. Decision making is a function which plays a crucial role in determining, managing, administering, settling, resolving or dissolving the conflict of trade-offs. Decision making is a process, which involves different stages. They are stating the problem, establishing the goals, identifying the alternatives, recognizing the criteria associated with the alternatives and categorizing the weights of each criterion.The process involved are choosing an appropriate decision making tools; evaluating the priorities over the alternatives along with the defined criteria; validating the obtained solution against the problem statement and implementing strategy. Decision makers are accountable for making important decisions.The decisionmaking process is simple when a firm or planning commission has a single option to choose. When there are many options the conflict will arisewhile choosing good and reliable optionconsidering all alternatives with its associated criterias'. Sometimes conflict may occur due to evaluating multiple criterias' in order to arrive at a compromise and more ideal solution. To overcome these issues various decision making methods have been developed and applied successfully in numerous real life instances.

Multi-Criteria Decision Making (MCDM): Multi-Criteria Decision Making (MCDM) is a decision making method in which the decision predominantly lies in choosing an alternative with optimum objective value, based on the evaluation of multiple criterion associated with each alternative.

Multi-Attribute Decision Analysis (MADA): Multi-Attribute Decision Analysis (MADA) is a decision making method which is very similar to MCDM, but the difference lies in referring "criterion" as "attribute".

Multi-Objective Decision Making/Analysis (MODM/MODA): Multi-Objective Decision Making is a decision making method which include choosing an alternative by evaluating its associated different criteria by focusing into more than one objective.

Multi-Attribute Utility Theory (MAUT): Multi-Attribute Utility Theory isa decision making method in which a decision maker has to decide between multiple alternatives (options) based on the different attributes (criteria) in which the whole system is found to be under uncertainty such as incomplete information. The merit of using MAUT is that it can process the information which is incomplete and uncertain directly into its decision support model.

Multi-Attribute Value Theory (MAVT): Multi-Attribute Value Theory is a decision making method which addresses the problem of selecting an alternative from a finite and discrete set of alternatives that have to be assessed on the basis of conflicting objectives. MAUT is considered to be the strong form of decision making than MAVT.

Decision Making Tools: Decision making tools such as Analytic Hierarchy Process (AHP), Analytic Network Process (ANP), Technique for the Order of prioritization by Similarity to Ideal Solution (TOPSIS), Outranking methods like ELECTRE and PROMETHEE, Grey Relational Analysis, Evidential Reasoning Approach, VIKOR and Fuzzy VIKOR are some of the successful techniques which have received a good attention by experts in solving complex systems.

2. Basic Definitions and Preliminaries:

Definition: Fuzzy Set [88] Let E be the universal set, let x be an element of E then the fuzzy subset $\underset{\sim}{A}$ of E is a set of ordered pairs. $\underset{\sim}{A} = \{(x \mid \mu_{\underset{\sim}{A}}(x))\}$, for all $x \in E$

Where,

(i) $\mu_{\underset{\sim}{A}}(x)$ is the grade (or) degree of membership of x in $\underset{\sim}{A}$.

(ii) $\mu_{\underset{\sim}{A}}(x)$ takes the value from the membership set M = [0,1] and

(iii) $\mu_{\underset{\sim}{A}}(x)$ is the membership function or characteristic function.

Definition: Zadeh's Extension Principle [89-91] Let *f* be a function such that $f : X \to Y$ and let $\underset{\sim}{A}$ be a fuzzy subset of X defined by, $\underset{\sim}{A} = \frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{x_2} + \frac{\mu_{\underset{\sim}{A}}(x_3)}{x_3} + \ldots + \frac{\mu_{\underset{\sim}{A}}(x_n)}{x_n}$

Zadeh's extension of f states that the image of fuzzy set $\underset{\sim}{A}$ under the mapping *f*(.) can be expressed as a fuzzy set $\underset{\sim}{B}$ such as, $\underset{\sim}{B} = \frac{\mu_{\underset{\sim}{A}}(x_1)}{y_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{y_2} + \frac{\mu_{\underset{\sim}{A}}(x_3)}{y_3} + \ldots + \frac{\mu_{\underset{\sim}{A}}(x_n)}{y_n}$ Where $y_i = f(x_i)$.

Definition: Interval Valued Fuzzy Sets [3] An interval valued fuzzy set $\underset{\sim}{A}$ (over a basic set *E*) is given by afunction $M_{\underset{\sim}{A}}(x)$ where $M_{\underset{\sim}{A}} : E \to [0,1]$, the set of all subintervals of the unitinterval, i.e. for every $x \in E, M_{\underset{\sim}{A}}(x)$ is an interval within [0, 1].

Definition: Intuitionistic Fuzzy Set [2] Let X is a nonempty set. An intuitionistic fuzzy set $\underset{\sim}{A}$ in X is an object having the form $\underset{\sim}{A} = \left\{ \left\langle x, \mu_{\underset{\sim}{A}}(x), \gamma_{\underset{\sim}{A}}(x) \right\rangle : x \in X \right\}$ with $\mu_{\underset{\sim}{A}}(x) : X \to [0,1]$ and $\gamma_{\underset{\sim}{A}}(x) : X \to [0,1]$, where,

(i) $\mu_{\underset{\sim}{A}}(x)$ represents the degree of membership

(ii) $\gamma_{\underset{\sim}{A}}(x)$ represents the degree of non-membership of the element $x \in X$ to the set $\underset{\sim}{A}$ where $\underset{\sim}{A} \subseteq X$, and for every element $x \in X$, $0 \le \mu_{\underset{\sim}{A}}(x) + \gamma_{\underset{\sim}{A}}(x) \le 1$.

Definition: Intuitionistic Fuzzy Index [2] The intuitionistic fuzzy index or hesitation margin of x in $\underset{\sim}{A}$ is defined by the formula, $\pi_{\underset{\sim}{A}}(x) = 1 - \mu_{\underset{\sim}{A}}(x) - \gamma_{\underset{\sim}{A}}(x)$ Where,

(i) $\pi_{\underset{\sim}{A}}(x)$ represents the degree of indeterminacy of $x \in X$ to the IFS $\underset{\sim}{A}$

(ii) $\mu_{\underset{\sim}{A}}(x)$ represents the degree of membership of $x \in X$ to the IFS $\underset{\sim}{A}$

(iii) $\gamma_{\underset{\sim}{A}}(x)$ represents the degree of non-membership of $x \in X$ to the IFS $\underset{\sim}{A}$

With $\pi_{\underset{\sim}{A}}(x) : X \to [0,1]$ and $0 \le \pi_{\underset{\sim}{A}}(x) \le 1$ for every $x \in X$.

Note: $\pi_{\underset{\sim}{A}}(x)$ expresses the lack of knowledge of whether x belongs to IFS $\underset{\sim}{A}$ or not.

Definition: Interval Valued Intuitionistic Fuzzy Sets [3] The map *f* assigns to every IVFS $\underset{\sim}{A}$ an IFS $\underset{\sim}{B} = f(\underset{\sim}{A})$ given by $\mu_{\underset{\sim}{B}}(x) = \inf M_{\underset{\sim}{A}}(x), \gamma_{\underset{\sim}{B}}(x) = \sup M_{\underset{\sim}{A}}(x)$

The map *g* assigns to every IFB $\underset{\sim}{B}$ an IVFS $M_{\underset{\sim}{A}}(x) = \left[\mu_{\underset{\sim}{B}}(x), 1 - \gamma_{\underset{\sim}{B}}(x) \right]$

The basic theory of IVIFS [3] Let a set *E* be fixed. An interval valued intuitionistic fuzzy sets (IVIFS) $\underset{\sim}{A}$ over *E* is an object having the form $\underset{\sim}{A} = \left\{ \left\langle x, M_{\underset{\sim}{A}}(x), N_{\underset{\sim}{A}}(x) \right\rangle : x \in E \right\}$

Where $M_{\underset{\sim}{A}}(x) \subseteq [0,1]$ and $N_{\underset{\sim}{A}}(x) \subseteq [0,1]$ are intervals and for every $x \in E$, $\sup M_{\underset{\sim}{A}}(x) + \sup N_{\underset{\sim}{A}}(x) \le 1$

Definition: Fuzzy Number [23, 29] A Fuzzy number $\underset{\sim}{A}$ is a fuzzy set on the real line, must satisfy the following conditions.

$(i)\ \mu_{\underset{\sim}{A}}(x_0)$ is piecewise continuous.

(ii) There exists atleast one $x_0 \in R$ with $\mu_{\underset{\sim}{A}}(x_0) = 1$.

$(iii)\ \underset{\sim}{A}$ must be normal and convex.

Definition: Triangular Fuzzy Number [23, 29, 64-65] A Triangular Fuzzy Number $\underset{\sim}{A}$ is a fuzzy subset of real line, whose membership function $\mu_{\underset{\sim}{A}}$ satisfies the following conditions:

$(i)\ \mu_{\underset{\sim}{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$.

$(ii)\ \mu_{\underset{\sim}{A}}(x) = 0$, where $-\infty < x \le a_1$.

$(iii)\ \mu_{\underset{\sim}{A}}(x)$ is strictly increasing with constant rate on $a_1 \le x \le b_1$.

$(iv)\ \mu_{\underset{\sim}{A}}(x) = 1$, where $x = b_1$.

$(v)\ \mu_{\underset{\sim}{A}}(x)$ is strictly decreasing with constant rate on $b_1 \le x \le c_1$.

$(vi)\ \mu_{\underset{\sim}{A}}(x) = 0$, where $c_1 \le x \le \infty$.

Triangular Fuzzy Number is defined as $\underset{\sim}{A} = (a_1, b_1, c_1)$, where all a_1, b_1, c_1 are real numbers and its membership function is given below.

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{(x-a_1)}{(b_1-a_1)} & for\ a_1 \le x \le b_1 \\ \dfrac{(c_1-x)}{(c_1-b_1)} & for\ b_1 \le x \le c_1 \\ 0 & for\ x > c_1 \end{cases}$$

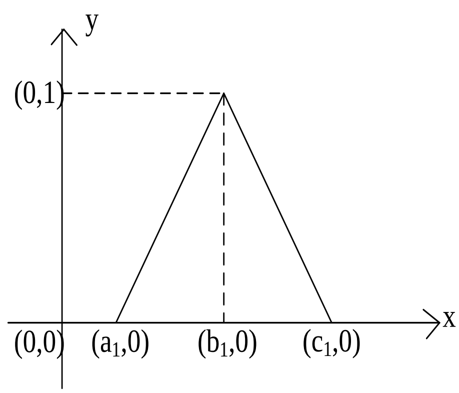

Figure 1: Triangular fuzzy number

Definition: Trapezoidal Fuzzy Number [29, 64-65] A Trapezoidal Fuzzy Number $\underset{\sim}{A}$ is a fuzzy subset of real line, whose membership function $\mu_{\underset{\sim}{A}}$ satisfies the following conditions:

$(i)\ \mu_{\underset{\sim}{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$.

$(ii)\ \mu_{\underset{\sim}{A}}(x) = 0$, where $-\infty < x \le a_1$.

$(iii)\ \mu_{\underset{\sim}{A}}(x)$ is strictly increasing with constant rate on $a_1 \le x \le b_1$.

$(iv)\ \mu_{\underset{\sim}{A}}(x) = 1$, where $b_1 \le x \le c_1$.

$(v)\ \mu_{\underset{\sim}{A}}(x)$ is strictly decreasing with constant rate on $c_1 \le x \le d_1$.

$(vi)\ \mu_{\underset{\sim}{A}}(x) = 0$, where $d_1 \le x \le \infty$.

A fuzzy set $\underset{\sim}{A}$ = {a, b, c, d} is said to trapezoidal fuzzy number if its membership function is given by where a ≤ b ≤ c ≤ d

$$\mu_{\underset{\sim}{A}}(x)=\begin{cases} 0 & for\ x<a_1 \\ \frac{(x-a_1)}{(b_1-a_1)} & for\ a_1\le x\le b_1 \\ 1 & for\ b_1\le x\le c_1 \\ \frac{(d_1-x)}{(d_1-c_1)} & for\ c_1\le x\le d_1 \\ 0 & for\ x>d_1 \end{cases}$$

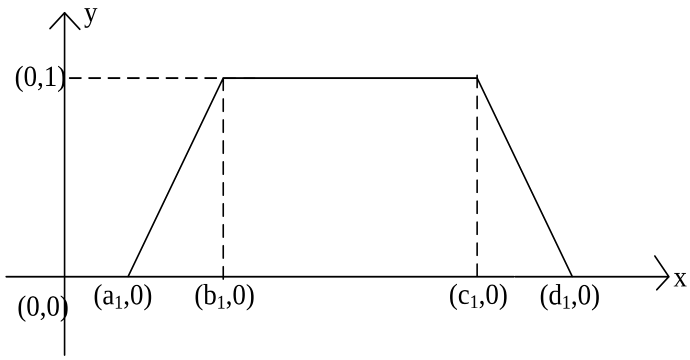

Figure 2: Trapezoidal fuzzy number

Definition: Hesitant Fuzzy Set [82] Let X be a fixed set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0,]. Mathematical representation of Hesitant fuzzy set: $\underset{\sim}{A}=\left\{\left\langle x,h_{\underset{\sim}{A}}(x)\right\rangle \mid x\in X\right\}$ Where $h_{\underset{\sim}{A}}(x)$ is a set of some values in [0, 1], denoting the possible membership degrees of the element *x*∈*X* to the set $\underset{\sim}{A}$.

3. Use of Formal Mathematical Concepts in Decision Making (5th Century – 19thCentury):

In Athens, Greece, during fifth century, decision making was done in two steps. First there would be a public oration; then people supporting the decision would raise their hands. Someone would count the number of hands gone up. Majority was decided in that manner. Further starting from 17th century in Economics [39], Commerce [39] and in other scientific fields [39], various philosophers [39], political leaders [39], business tycoons [39], mathematicians [39], had introduced many decision modelsand decision theories [39] in order to assist the standard of living and meet the problematic situationsthat happened in and around the situations.

Concepts from 17th Century: In 17th century, Benjamin Franklin introduced paper system for deciding his opinion on an important issue.Marquis de Condorcet in the year 1973 devised the three stages for decision making [39], which are:

Stage-1: The first stage of the decision theory is todiscuss the principles that will serve as basis for decision in a general issue. Various aspects and effects of issuesare examined.

Stage-2: The opinions collected from the stage 1 are combined and the importance of each alternative becomes clearer

Stage-3: The tangible option is made from the set of alternatives.

Concepts from 19th Century: Francis Edgeworth [39] of mid 19th century was referred to be one of the significant developers of neoclassical economics. He was the first person to adopt certain mathematical concepts in decision making. Vilfredo Pareto [39]of late 19th century was the first to mathematically study the aggregation of conflicting criteria. He introduced the concept of efficiency (Pareto-optimality) which is a prominent concept in modern MCDM theory.Earlier foundations in VIKOR model was based on the Pareto-optimality condition.

Pareto Optimality and Ophelimity (Mid 20th Century) [47]: The concept of Pareto optimality evolved in the economic equilibrium and welfare theories, which is based on the economic satisfaction. Dinh Thế Lucandothers [47] discussed the concept of the theory of Pareto optimality. Kuhn and Tucker [41] in the year 1951 established the necessary and sufficient conditions for optimality. Koopmans [40] introduced the concept of Pareto optimality in the field of operations research.LotfiZadeh in the year 1963 [92] discussed the optimality and non-scalar-valued performance criteria. The earlier work of Klinger in the year 1958 [38] attributed the vector-valued performance criteria. Additional developments in the field of optimality are made by the economists DaCunha-Polak [39], Geoffrion[39], J. M. Browein[13], D. Zhuang[14] in the middle of the 20th century.

Group Rationality or Pareto Optimality or Pareto Efficiency [47]: Group rationality is the general allocation of cost ought to be given to the players which must be equal to the total cost of the game.

$$\sum_{i \in N} y_i = c(N)$$

Where y_i is the cost allocated to player i and *c(N)* is the total cost of the game.

Individual rationality [47]: Players act efficiently when theyform a coalition. There must be an acceptable distribution of the payoff. Distribution where a player receives less than what he/she could obtain on his/her own, without cooperating with anyone else is unacceptable. This condition is known as individual rationality. Individual rationality is the term which refers to the cost allocated to each player and the cost should not exceedthe amount a player would have to pay if he acted without the others.

$$y_i \geq c\left(\{i\}\right), \forall i \in N$$

Where y_i is the cost allocated to player i and $c\left(\{i\}\right)$ is the cost of the player if he acted without others.

4. Political and Economic Decisions (Early 20th Century)

In 1910, John Dewey developed the theory of decision making in five consecutive stages [39]. Later, Herbert Simon in the year 1960 modified the context of stages which was explained by Dewey according to the organizational set up. Frank. P. Ramsey introduced expected utility model and laid the foundations for MCDM methods.In the early 19th century, the decision makers developed successful theoretical decision making concepts to predict the social choice of an individual and of a group. Political decisions and Economic decisions [1] flourished in the early 19th century to mid 19th century with the theoretical concepts introduced by the researchers Jeremy Bentham, N. Kaldor, J. R. Hicks, L. Robbins, Kenneth J. Arrow, von Neumann, C. West Churchman, Russell. L. Ackoff and others. Jeremy Bentham presented the theory of interpersonal comparisons of utility and his work was argued by Economists N. Kaldor [1], J. R. Hicks [1] and L. Robbins [1]. Kenneth J. Arrow [1] initiated the work on social choices in the year 1948 and established various aspects and developments in political and economic decisions in the book of "*Social Choice and Individual Values*". Von Neumann and Morgenstern [1] introduced the concept of measurable utility and argued that without measurable utility, interpersonal comparisons of utility has no meaning. In early 1950s, it was noted that the existing decision making methods were technically insufficient in processing conflicting and contradictory information.In 1952, Richard Bellman [10-11] stated the theory of dynamic programming in order to study the multistage process. Further he developed and described the dynamic programming with the help of an application [9, 12]. Two of his articles publishedin the year 1954 titled Decision-Making in the face of Uncertainty - I [7] and II [8] revealed the problem of simple multi-stage decisionmaking process. Those papers showed that an approximate solution of plausible and intuitive sort could be obtained under certain reasonable assumptions. Also in the subsequent work, he considered a more difficult class of problems, involving conflict between two groups. He noticed that, when two groups are in direct conflict, zero-sum games will be the right choice for making a decision. But when two groups are partly opposed, then non-zero sum games are the wise way to formulate and solve the system. Richard Bellman along with Lotfi A. Zadeh [6] in the year 1970 focused on the decision making in a fuzzy environment. They proposed a theory using Max-min composition rule to solve the multi-stage system where the system is bound to have uncertain conditions and incomplete information.

Compromise Programming (1970s – 1990s): The idea of compromise solution was introduced in MCDM by P.L.Yu [85-86] and by Milan Zeleny [95-99] in 1973. The objective is to produce a solution that is closest to the 'ideal' solution which is measured in terms of comparing distances of various points to a reference point.

Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR) (1990s – 2000s): The literal meaning of Vise Kriterijumska Optimizacija I Kompromisno Resenje is Multi-criteria Optimization and Compromise Solution. In1979, S. Opricovic [24] developed the basic idea of VIKOR in his Ph.D. dissertation. Earlier P.L. Yu developed compromise solution in battling with conflict criteria. Later in 1980, Lucien Duckstein presented an application with the basic ideas of VIKOR in his article, 'Multi objective Optimization in River Basin Development' [24]. The name VIKOR appeared for the first time in 1990 in the article titled 'Program skipaket VIKOR zavisekriterijumskokompromisnorangiranje' by S. Opricovic [57-61] as a compromising application technique to implement within MCDM. In 1994, The VIKOR algorithm determines the weight stability intervals for the obtained compromise solution with the input weights, given by the experts. In 1998, S. Opricovic introduced the Multicriteria index based on the particular measure of 'closeness' to the 'ideal' solution. Compromise programming wasapplied to the water resources system in the central Tisza river basin. The measure L_{pj} was introduced by Duckstein and Opricovic in the year 1980 [24] to represent the distance of the alternative A_j from the best ideal solution. Basically it is a relative measure. The multi-criteria measure for compromise ranking is developed from the L_p - metric. It was used asan aggregating function which is the measure of closeness in a compromised programming method (Yu, 1973; Zeleny, 1982) [86, 95-99].

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \cdots & C_n \end{matrix} \\ \begin{pmatrix} f_{11} & f_{12} & f_{13} & \cdots & f_{1n} \\ f_{21} & f_{22} & f_{23} & \cdots & f_{2n} \\ f_{31} & f_{32} & f_{33} & \cdots & f_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & f_{m3} & \cdots & f_{mn} \end{pmatrix} \end{array}$$

The compromise programming method was developed to perform multi-criteria optimization and by reducing the set of noninferior (Pareto optimal) solutions.

Comparative Analysis of VIKOR and TOPSIS: In 2004, Serafim Opricovic and Gwo-Hshiung Tzeng [59] compared the compromise solution obtained from VIKOR and TOPSIS techniques with a numerical illustration.Further they combined traditional VIKOR method with 2-tuple linguistic method in the year 2013 [50]. Y. Ju and A. Wang [84] extended VIKOR method for multi-criteria group decision making problem with linguistic information. They proposed a new method by determining the criteria weights through transforming linguistic weights into trapezoidal fuzzy numbers and then defuzzifying the trapezoidal fuzzy numbers to crisp numbers.

Comparative Analysis of Extended VIKOR with Outranking Methods: Serafim Opricovic and Gwo-Hshiung Tzengin the year 2007 [60] extended the VIKOR method with a stability analysis and compared the obtained results with the other existing methods like TOPSIS, PROMETHEE and ELECTRE.

VIKOR with Interval Numbers: The interval numbers are the simplest form of representing incomplete information by considering the value of the parameter (criteria) within the interval. In 2009 Sayadi, et.al.,[75] extended VIKOR method for decision making problem with interval numbers by adapting the entries of decision matrix by interval number which is given in the below decision making matrix.

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & \cdots & C_n \end{matrix} \\ \begin{pmatrix} [f_{11}^L, f_{11}^U] & [f_{12}^L, f_{12}^U] & [f_{13}^L, f_{13}^U] & \cdots & [f_{1n}^L, f_{1n}^U] \\ [f_{21}^L, f_{21}^U] & [f_{22}^L, f_{22}^U] & [f_{23}^L, f_{23}^U] & \cdots & [f_{2n}^L, f_{2n}^U] \\ [f_{31}^L, f_{31}^U] & [f_{32}^L, f_{32}^U] & [f_{33}^L, f_{33}^U] & \cdots & [f_{3n}^L, f_{3n}^U] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [f_{m1}^L, f_{m1}^U] & [f_{m2}^L, f_{m2}^U] & [f_{m3}^L, f_{m3}^U] & \cdots & [f_{mn}^L, f_{mn}^U] \end{pmatrix} \end{array}$$

VIKOR with Interval-Valued Fuzzy Sets: In order to describe the subjective opinion in a more accurate way, the interval-valued fuzzy set had been used as a natural extension of fuzzy sets [89-91]. Behnam Vahdani and others [83] extended the VIKOR method based on interval-valued fuzzy sets in the year 2010 to represent the expert opinion in a more accurate way by describing the performance rating value in interval-valued fuzzy sets. Behnam Vahdani et, al., [83] developed decision matrix (DM) with the interval valued fuzzy set entries based on the performance rating value of each alternative over the criteria.

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \cdots & C_n \end{matrix} \\ \begin{pmatrix} \left[\left(\underset{\sim}{x}_{11}^{(1)}, \underset{\sim}{x}_{11}^{(1)'}\right); \underset{\sim}{x}_{11}^{(2)}; \left(\underset{\sim}{x}_{11}^{(3)}, \underset{\sim}{x}_{11}^{(3)'}\right)\right] & \left[\left(\underset{\sim}{x}_{12}^{(1)}, \underset{\sim}{x}_{12}^{(1)'}\right); \underset{\sim}{x}_{12}^{(2)}; \left(\underset{\sim}{x}_{12}^{(3)}, \underset{\sim}{x}_{12}^{(3)'}\right)\right] & \cdots & \left[\left(\underset{\sim}{x}_{1n}^{(1)}, \underset{\sim}{x}_{1n}^{(1)'}\right); \underset{\sim}{x}_{1n}^{(2)}; \left(\underset{\sim}{x}_{1n}^{(3)}, \underset{\sim}{x}_{1n}^{(3)'}\right)\right] \\ \left[\left(\underset{\sim}{x}_{21}^{(1)}, \underset{\sim}{x}_{21}^{(1)'}\right); \underset{\sim}{x}_{21}^{(2)}; \left(\underset{\sim}{x}_{21}^{(3)}, \underset{\sim}{x}_{21}^{(3)'}\right)\right] & \left[\left(\underset{\sim}{x}_{22}^{(1)}, \underset{\sim}{x}_{22}^{(1)'}\right); \underset{\sim}{x}_{22}^{(2)}; \left(\underset{\sim}{x}_{22}^{(3)}, \underset{\sim}{x}_{22}^{(3)'}\right)\right] & \cdots & \left[\left(\underset{\sim}{x}_{2n}^{(1)}, \underset{\sim}{x}_{2n}^{(1)'}\right); \underset{\sim}{x}_{2n}^{(2)}; \left(\underset{\sim}{x}_{2n}^{(3)}, \underset{\sim}{x}_{2n}^{(3)'}\right)\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\left(\underset{\sim}{x}_{m1}^{(1)}, \underset{\sim}{x}_{m1}^{(1)'}\right); \underset{\sim}{x}_{m1}^{(2)}; \left(\underset{\sim}{x}_{m1}^{(3)}, \underset{\sim}{x}_{m1}^{(3)'}\right)\right] & \left[\left(\underset{\sim}{x}_{m2}^{(1)}, \underset{\sim}{x}_{m2}^{(1)'}\right); \underset{\sim}{x}_{m2}^{(2)}; \left(\underset{\sim}{x}_{m2}^{(3)}, \underset{\sim}{x}_{m2}^{(3)'}\right)\right] & \cdots & \left[\left(\underset{\sim}{x}_{mn}^{(1)}, \underset{\sim}{x}_{mn}^{(1)'}\right); \underset{\sim}{x}_{mn}^{(2)}; \left(\underset{\sim}{x}_{mn}^{(3)}, \underset{\sim}{x}_{mn}^{(3)'}\right)\right] \end{pmatrix} \end{array}$$

Where $\left[\left(\tilde{x}_{11}^{(1)}, x_{11}^{(1)'}\right); \tilde{x}_{11}^{(2)}; \left(\tilde{x}_{11}^{(3)}, \tilde{x}_{11}^{(3)'}\right)\right]$ represent the performance rating of alternative A_1 over the criteria C_1, which has the form of triangular fuzzy number $[\tilde{l}_{11}, \tilde{m}_{11}, \tilde{n}_{11}]$ with the use of interval value numbers $\left(\tilde{x}_{11}^{(1)}, x_{11}^{(1)'}\right)$ and $\left(\tilde{x}_{11}^{(3)}, \tilde{x}_{11}^{(3)'}\right)$ in the left $(\tilde{l}_{11})$ and right $(\tilde{n}_{11})$ values, which determines the lower and upper bound of the numerical rating shows more accuracy when compared to the ordinary fuzzy set.

Modified VIKOR Method: Chia-Ling Chang [16] proposed a modified VIKOR method in order to reduce the numerical difficulty that emerged through the traditional VIKOR in processing a performance rating of an alternative over various parameters in the year 2010. Further, Chia-Ling Chang experimentally verified the usefulness of modified VIKOR by producing the compromise solution with high acceptance [17] which is closest to the optimal solution. Whereas the traditional VIKOR method sometimes failed to produce a VIKOR index Q_j. In an additional work of Chia-Ling Chang and Chung-Hsin Hsu [17] in the year 2011, classification of land subdivisions according to the watershed vulnerability had been made with the help of modified VIKOR method. Also they applied VIKOR method for prioritizing land-use restraint strategies in the Tseng-Wen reservoir watershed in the year 2009 [18].

VIKOR with Interval-Valued Intuitionistic Fuzzy Sets: Jin Han Park et, al., in the year 2011 [62] proposed an extension of VIKOR basedon interval-valued intuitionistic fuzzy set in which all the performance rating entries in the decision matrix (DM) are characterized by interval-valued intuitionistic fuzzy sets. The decision matrix (DM) proposed by Jin Han Park et, al., is described as follows:

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \left[\left(\tilde{a}_{11}^{(k)}, \tilde{b}_{11}^{(k)}\right)\left(\tilde{c}_{11}^{(k)}, \tilde{d}_{11}^{(k)}\right)\right] & \left[\left(\tilde{a}_{12}^{(k)}, \tilde{b}_{12}^{(k)}\right)\left(\tilde{c}_{12}^{(k)}, \tilde{d}_{12}^{(k)}\right)\right] & \cdots & \left[\left(\tilde{a}_{1n}^{(k)}, \tilde{b}_{1n}^{(k)}\right)\left(\tilde{c}_{1n}^{(k)}, \tilde{d}_{1n}^{(k)}\right)\right] \\ \left[\left(\tilde{a}_{21}^{(k)}, \tilde{b}_{21}^{(k)}\right)\left(\tilde{c}_{21}^{(k)}, \tilde{d}_{21}^{(k)}\right)\right] & \left[\left(\tilde{a}_{22}^{(k)}, \tilde{b}_{22}^{(k)}\right)\left(\tilde{c}_{22}^{(k)}, \tilde{d}_{22}^{(k)}\right)\right] & \cdots & \left[\left(\tilde{a}_{2n}^{(k)}, \tilde{b}_{2n}^{(k)}\right)\left(\tilde{c}_{2n}^{(k)}, \tilde{d}_{2n}^{(k)}\right)\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\left(\tilde{a}_{m1}^{(k)}, \tilde{b}_{m1}^{(k)}\right)\left(\tilde{c}_{m1}^{(k)}, \tilde{d}_{m1}^{(k)}\right)\right] & \left[\left(\tilde{a}_{m2}^{(k)}, \tilde{b}_{m2}^{(k)}\right)\left(\tilde{c}_{m2}^{(k)}, \tilde{d}_{m2}^{(k)}\right)\right] & \cdots & \left[\left(\tilde{a}_{mn}^{(k)}, \tilde{b}_{mn}^{(k)}\right)\left(\tilde{c}_{mn}^{(k)}, \tilde{d}_{mn}^{(k)}\right)\right] \end{pmatrix} \end{array}$$

Where, $\left[\left(\tilde{a}_{11}^{(k)}, \tilde{b}_{11}^{(k)}\right)\left(\tilde{c}_{11}^{(k)}, \tilde{d}_{11}^{(k)}\right)\right]$ represents the interval-valued intuitionistic fuzzy set with $\left(\tilde{a}_{11}^{(k)}, \tilde{b}_{11}^{(k)}\right)$ which indicates the degree that the alternative A_1 satisfy the criteria C_1. $\left(\tilde{c}_{11}^{(k)}, \tilde{d}_{11}^{(k)}\right)$ indicates the degree of not satisfying of the alternative A_1 over the criteria C_1.

VIKOR with Dynamic Intuitionistic Fuzzy Sets: In 2011 Jin Han Park and others [62] extended the VIKOR method to dynamic intuitionistic fuzzy sets and presented two aggregation operators called dynamic intuitionistic fuzzy weighted geometric operator and uncertain dynamic intuitionistic fuzzy weighted geometric operator.

Fuzzy VIKOR with Fuzzy Number: Amir Sanayei [74] and others in the year 2010 used linguistic values to assess the ratings and weights of the various factors associated with the decision making problem. Also he employed trapezoidal fuzzy number and triangular fuzzy number to express the linguistic ratings of each parameter in the decision making problem. In 2011, S. Opricovic [57], faculty of civil engineering illustrated an application of fuzzy VIKOR method to water resources planning, where he used entries of decision matrix (DM) as the triangular fuzzy number to handle the incompleteinformation.

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{pmatrix} (l_{11}, m_{11}, r_{11}) & (l_{12}, m_{12}, r_{12}) & (l_{13}, m_{13}, r_{13}) & \cdots & (l_{1n}, m_{1n}, r_{1n}) \\ (l_{21}, m_{21}, r_{21}) & (l_{22}, m_{22}, r_{22}) & (l_{23}, m_{23}, r_{23}) & \cdots & (l_{2n}, m_{2n}, r_{2n}) \\ (l_{31}, m_{31}, r_{31}) & (l_{32}, m_{32}, r_{32}) & (l_{33}, m_{33}, r_{33}) & \cdots & (l_{3n}, m_{3n}, r_{3n}) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (l_{m1}, m_{m1}, r_{m1}) & (l_{m2}, m_{m2}, r_{m2}) & (l_{m3}, m_{m3}, r_{m3}) & \cdots & (l_{mn}, m_{mn}, r_{mn}) \end{pmatrix} \end{array}$$

Ali Shemshadiet. al., [76] in the year 2011 developed an application for supplier selection based on the subjective opinion obtained by the decision makers' and the opinions are processed by converting them into a trapezoidal fuzzy number.

$$DM = \begin{matrix} & C_1 & C_2 & C_3 & \cdots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \left(\begin{matrix} (a_{11},b_{11},c_{11},d_{11}) \\ (a_{21},b_{21},c_{21},d_{21}) \\ (a_{31},b_{31},c_{31},d_{31}) \\ \vdots \\ (a_{m1},b_{m1},c_{m1},d_{m1}) \end{matrix} \right. & \begin{matrix} (a_{12},b_{12},c_{12},d_{12}) \\ (a_{22},b_{22},c_{22},d_{22}) \\ (a_{32},b_{32},c_{32},d_{32}) \\ \vdots \\ (a_{m2},b_{m2},c_{m2},d_{m2}) \end{matrix} & \begin{matrix} (a_{13},b_{13},c_{13},d_{13}) \\ (a_{23},b_{23},c_{23},d_{23}) \\ (a_{33},b_{33},c_{33},d_{33}) \\ \vdots \\ (a_{m3},b_{m3},c_{m3},d_{m3}) \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left. \begin{matrix} (a_{1n},b_{1n},c_{1n},d_{1n}) \\ (a_{2n},b_{2n},c_{2n},d_{2n}) \\ (a_{3n},b_{3n},c_{3n},d_{3n}) \\ \vdots \\ (a_{mn},b_{mn},c_{mn},d_{mn}) \end{matrix} \right) \end{matrix}$$

In the above decision matrix all the entries are trapezoidal fuzzy number with A_i and C_i represents alternative and criteria respectively.

Fuzzy VIKOR with AHP: Tolga Kaya and Cengiz Kahraman [35] proposed an integrated VIKOR-AHP methodology to make a selection among the alternative forestation areas in Istanbul. The weights of the criteria associated with the alternative are calculated by fuzzy pair-wise comparison matrices of AHP. Forestry decision making is a multifacetedproblem which involves a process of balancing diverse ecological, social, and economic aspects over space and timewhich are hard to quantify. The decision matrix (DM) adapted by them [35]involves entries comprising the opinion of N decision makers over each alternative which is represented with triangular fuzzy number.

$$DM = \begin{matrix} & C_1 & C_2 & C_3 & \cdots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \left(\begin{matrix} \underset{\sim}{x}_{11} \\ \underset{\sim}{x}_{21} \\ \underset{\sim}{x}_{31} \\ \vdots \\ \underset{\sim}{x}_{m1} \end{matrix} \right. & \begin{matrix} \underset{\sim}{x}_{12} \\ \underset{\sim}{x}_{22} \\ \underset{\sim}{x}_{32} \\ \vdots \\ \underset{\sim}{x}_{m2} \end{matrix} & \begin{matrix} \underset{\sim}{x}_{13} \\ \underset{\sim}{x}_{23} \\ \underset{\sim}{x}_{33} \\ \vdots \\ \underset{\sim}{x}_{m2} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \ddots \\ \cdots \end{matrix} & \left. \begin{matrix} \underset{\sim}{x}_{1n} \\ \underset{\sim}{x}_{2n} \\ \underset{\sim}{x}_{3n} \\ \vdots \\ \underset{\sim}{x}_{mn} \end{matrix} \right) \end{matrix}$$

Where, $\underset{\sim}{x}_{11} = \frac{1}{N}\left[\underset{\sim}{x}_{11}^{1} + \underset{\sim}{x}_{11}^{2} + \underset{\sim}{x}_{11}^{3} + \ldots + \underset{\sim}{x}_{11}^{N}\right]$ is the equation employed for finding the rating of the each alternative with respect to the criteria associated with them and N represents the number of decision maker involved in providing the opinion over the alternatives. In the year 2011, J.R. San Cristobal [21] used VIKOR-AHP method in the selection of a renewable energy project corresponding to the Renewable Energy Plan launched by the Spanish Government in Spain. Also, Peyman Mohammady and Amin Amid [53] have presented a decision making tool in the same year, by integrating Fuzzy AHP with Fuzzy VIKOR for supplier selection in an Agile and Modular Virtual Enterprise.

VIKOR with GRA Techniques: Ming-Shin Kuo and Gin-Shuh Liang [42] in the year 2011 combined Grey Relational Analysis (GRA) with VIKOR method to deal with the evaluation of service quality of Northeast-Asian International Airports by performing customer surveys. Grey relational analysis can be used to show the correlation between the preferred factors and other compared factors (alternative) of a system. The decision matrix inputs were taken to be the linguistic values expressed in terms of triangular fuzzy numbers through the subjective opinion of the N decision makers as mentioned in the above section [42]. The decision matrix $\underset{\sim}{D}$ is given as follows;

$$\underset{\sim}{D} = \left[\underset{\sim}{r}_{ij}\right]_{m\times n} \text{ With } \underset{\sim}{r}_{ij} = \left(\frac{l_{ij}}{r_j^*}, \frac{m_{ij}}{r_j^*}, \frac{r_{ij}}{r_j^*}\right); j \in B \text{ and } r_j^* = \max_i r_{ij} \text{ if } j \in B$$

$\underset{\sim}{r}_{ij} = \left(\frac{l_j^-}{r_{ij}}, \frac{l_j^-}{m_{ij}}, \frac{l_j^-}{l_{ij}}\right); j \in C$ and $l_j^- = \min_i l_{ij}$ if $j \in C$, where B represents the Benefit Criteria and C represents the Cost Criteria respectively. VIKOR with GRA technique assumes positive ideal and negative ideal solutions as preferred factors and each of the alternatives as compared factors to attain the weighted grey relation coefficient. The positive and negative ideal solution is given by $A* = \left[\underset{\sim}{r}_{01}^*, \underset{\sim}{r}_{02}^*, \underset{\sim}{r}_{03}^*, \ldots \underset{\sim}{r}_{0n}^*\right]; A^- = \left[\underset{\sim}{r}_{01}^-, \underset{\sim}{r}_{02}^-, \underset{\sim}{r}_{03}^-, \ldots \underset{\sim}{r}_{0n}^-\right]$ where $\underset{\sim}{r}_{0j}^* = \max_i\left(\underset{\sim}{r}_{ij}\right)$; $\underset{\sim}{r}_{0j}^- = \min_i\left(\underset{\sim}{r}_{ij}\right)$ for all j=1,2,…n. Then the weighted grey relation coefficient is given by the formula;

$$\gamma\left(\tilde{r}_{0j}^{u},\tilde{r}_{ij}\right)=\frac{\min_i \min_j \tilde{d}_{ij}^{wu}+\xi \max_i \max_j \tilde{d}_{ij}^{wu}}{\tilde{d}_{ij}^{u}+\xi \max_i \max_j \tilde{d}_{ij}^{u}}$$

Where $\tilde{d}_{ij}^{wu}=d\left(\tilde{w}_j\tilde{r}_{0j}^{u},\tilde{w}_j\tilde{r}_{ij}\right)$; $\tilde{d}_{ij}^{u}=d\left(\tilde{r}_{0j}^{u},\tilde{r}_{ij}\right)$; $u=*,-$ and ξ is the resolving coefficient $\xi\in[0,1]$.

VIKOR in Intuitionistic Fuzzy Environment: VIKOR method with the extension of intuitionistic fuzzy environment was developed by Kavita Devi [22] in the year 2011. The entriesin the decision matrix, i.e., the performance rating values of each alternative with respect to the criteria has been expressed by the triangular intuitionsitic fuzzy sets and it is given by;

$$DM=\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{pmatrix} \left\langle\begin{matrix}[(l_{11},m_{11},r_{11}),\mu_{\tilde A}(x)]; \\ [(l_{11}^1,m_{11}^1,r_{11}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{12},m_{12},r_{12}),\mu_{\tilde A}(x)]; \\ [(l_{12}^1,m_{12}^1,r_{12}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{13},m_{13},r_{13}),\mu_{\tilde A}(x)]; \\ [(l_{13}^1,m_{13}^1,r_{13}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}[(l_{1n},m_{1n},r_{1n}),\mu_{\tilde A}(x)]; \\ [(l_{1n}^1,m_{1n}^1,r_{1n}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle \\ \left\langle\begin{matrix}[(l_{21},m_{21},r_{21}),\mu_{\tilde A}(x)]; \\ [(l_{21}^1,m_{21}^1,r_{21}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{22},m_{22},r_{22}),\mu_{\tilde A}(x)]; \\ [(l_{22}^1,m_{22}^1,r_{22}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{23},m_{23},r_{23}),\mu_{\tilde A}(x)]; \\ [(l_{23}^1,m_{23}^1,r_{23}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}[(l_{2n},m_{2n},r_{2n}),\mu_{\tilde A}(x)]; \\ [(l_{2n}^1,m_{2n}^1,r_{2n}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle \\ \left\langle\begin{matrix}[(l_{31},m_{31},r_{31}),\mu_{\tilde A}(x)]; \\ [(l_{31}^1,m_{31}^1,r_{31}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{32},m_{32},r_{32}),\mu_{\tilde A}(x)]; \\ [(l_{32}^1,m_{32}^1,r_{32}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{33},m_{33},r_{33}),\mu_{\tilde A}(x)]; \\ [(l_{33}^1,m_{33}^1,r_{33}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}[(l_{3n},m_{3n},r_{3n}),\mu_{\tilde A}(x)]; \\ [(l_{3n}^1,m_{3n}^1,r_{3n}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left\langle\begin{matrix}[(l_{m1},m_{m1},r_{m1}),\mu_{\tilde A}(x)]; \\ [(l_{m1}^1,m_{m1}^1,r_{m1}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{m2},m_{m2},r_{m2}),\mu_{\tilde A}(x)]; \\ [(l_{m2}^1,m_{m2}^1,r_{m2}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \left\langle\begin{matrix}[(l_{m3},m_{m3},r_{m3}),\mu_{\tilde A}(x)]; \\ [(l_{m3}^1,m_{m3}^1,r_{m3}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle & \cdots & \left\langle\begin{matrix}[(l_{mn},m_{mn},r_{mn}),\mu_{\tilde A}(x)]; \\ [(l_{mn}^1,m_{mn}^1,r_{mn}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle \end{pmatrix}\end{array}$$

Where, $\left\langle\begin{matrix}[(l_{11},m_{11},r_{11}),\mu_{\tilde A}(x)]; \\ [(l_{11}^1,m_{11}^1,r_{11}^1),\gamma_{\tilde A}(x)]\end{matrix}\right\rangle$ represents the performance rating of alternative A_1 with the criteria C_1 , $\mu_{\tilde A}(x)$ and $\gamma_{\tilde A}(x)$ represents the triangular membership and non-membership values of the performance rating.

Comprehensive VIKOR Method: In the year 2011, Ali Jahanet, al., proposed a comprehensive version of VIKOR method [33] in order to avoid the numerical difficulties in choosing the performance rating between the alternatives with respect to the criterias' and also it covers all objectives in MCDM. Bahraminasab and Ali Jahan applied the comprehensive VIKOR method [4] for selecting the best femoral component material among the set of alternatives of total knee replacement in the year 2011.

Dematel Based ANP (DANP) with VIKOR: DEMATEL is the tool to validate the cause and effect relationship among variables/criteria in a system which has been widely applied in many situations such as marketing strategies, e-learning evaluation, control systems and safety problems. In 2012, Yung-Lan Wang and Gwo-Hshiung Tzeng combined DEMATEL technique with Analytic Network Process (ANP) and VIKOR method for clarifying the interrelated relationships of brand marketing and creating a brand value by evaluating the customer's satisfaction over the brand in Taiwan [84]. In addition to this, C - H. Hsu, Fu-Kwun Wang and Qwo-Hshiung Tzeng used DEMATEL based ANP-VIKOR model for the vendor selection problem in the year 2012. Later in the year 2013, Kao-Yi Shen, Min-Ren Yan and Gwo-Hshiung Tzeng applied VIKOR-DEMATEL based ANP (DANP) for glamor stock selection problem.

VIKOR with Hesitant Fuzzy Sets: Hesitancy fuzzy set (HFS) [82] was introduced by Torra and Narukawa in the year 2009to encourage the decision maker (experts) in choosing the membership degree of an element from several possible values between 0 and 1 without any hesitation. In the year 2013, HuchangLaio and Zeshui Xu developed a multi-criteria decision making theory by adapting hesitant fuzzy information in the VIKOR method [43]. The decision matrix (DM) developed by them based on the hesitant fuzzy information is illustrated as follows;

$$DM=\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{pmatrix} \tilde{h}_{11} & \tilde{h}_{12} & \cdots & \tilde{h}_{1n} \\ \tilde{h}_{21} & \tilde{h}_{22} & \cdots & \tilde{h}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{h}_{m1} & \tilde{h}_{m2} & \cdots & \tilde{h}_{mn} \end{pmatrix}\end{array}$$

Where $\underset{\sim}{h}_{11}$ represents the hesitant fuzzy element (Definition 2.11) defined on the performance rating of alternative A_1 over the criteria C_1.

Fuzzy ELECTRE with VIKOR: In 2013, Zandi and Roghanian extended the Fuzzy ELECTRE outranking method with VIKOR to rank a set of alternatives versus a set of criteria to show the expert preferences [93].

Induced Aggregation Operator with VIKOR: In the year 2013, Hu-Chen Liu et, al., developed a method of using induced ordered aggregation operator with VIKOR method [45] for analyzing the complex attitudinal character of the decision maker and provide much more complete information for decision making.

Fuzzy VIKOR with Failure Mode and Effect Analysis (FMEA): In the year 2014, Hossein Safariand others used Failure Mode and Effect Analysis (FMEA) [73] for evaluating enterprise architecture risks with the extension of fuzzy VIKOR to prioritize the risk factors.

Sentiment Analysis with VIKOR: Daekook Kang and Yongtae Park [34] presented a new framework for measurement if customer satisfaction in mobile service by combining VIKOR approach with the "Sentiment analysis".

5. Conclusion:

This paper presents several applications and concepts, which illustrate the use of VIKOR MCDM technique and some of the recently developed theoretical extensions. This review work covers the major conceptual developments that took place in the application of VIKOR method from the year 1973 to 2015. Also the background history of MCDM methods dated back from 5th century to 19th century is briefly given. Further the theoretical extension and modification of VIKOR method based on the other decision making methods have been clearly explained.

6. References:

1. Arrow, K. J., "Social Choice and Individual Values", Cowles Foundation for Research in Economics at Yale University, John Wiley and Sons, Inc., U.S.A, 1963.
2. Atanassov, K. T., "Intuitionistic Fuzzy Sets", Fuzzy Sets and Systems, Elsevier Science Publications, 20(1), pp. 87-96, 1986.
3. Atanassov, K. T., and Gargov, G., "Interval Valued Intuitionistic Fuzzy Sets", Fuzzy Sets and Systems, 31, pp. 343-349, 1989.
4. Bahraminasab, M., and Jahan, A., "Material selection for femoral component of total knee replacement using comprehensive VIKOR", International Journal of Materials and Design, Elsevier Science Publishers, 32, pp. 4471-4477, 2011.
5. Beinat, E., and Nijkamp, P., (Eds) "Multicriteria Analysis for Land-Use Management", Springer-Science+Buisness Media, B.V, Dordrecht, 1998.
6. Bellman, R. E, and Zadeh, L. A., "Decision Making in a Fuzzy Environment", Management Science, 17, pp. 141-164, 1970.
7. Bellman, R. E., "Decision-Making in the face of Uncertainty - I", Naval Research Logistics Quarterly, 1, pp. 230-232, 1954.
8. Bellman, R. E., "Decision-Making in the face of Uncertainty - II", Naval Research Logistics Quarterly, 1, pp. 327-332, 1954.
9. Bellman, R. E., "On Some Applications of the Theory of Dynamic Programming to Logistics", Naval Research Logistics Quarterly, 1, pp. 141-153, 1954.
10. Bellman, R. E., "On the Theory of Dynamic Programming", Proceedings of the National Academy of Sciences, 38(8), pp. 716-719, 1952.
11. Bellman, R. E., "The Theory of Dynamic Programming", Bulletin of American Mathematical Society, 60, pp. 503-516, 1954.
12. Bellman, R. E., and Zadeh, L. A., "Some Problems in the Theory of Dynamic Programming", Econometrica, 22(1), pp. 37-48, 1954.
13. Borwein, J. M., "The Geometry of Pareto Efficiency over Cones", MathematischeOperationsforchung und Statistik, Serie Optimization, 11, pp. 235-248, 1980.
14. Borwien, J. M., and Zhuang, D., "Super Efficiency in Vector Optimization", Transactions of the American Mathematical Society, 339, pp. 105-122, 1993.
15. Cai, X., Lasdon, L., and Michelsen, A. M., "Group Decision Making in Water Resources Planning Using Multiple Objective Anaysis", Journal of Water Resources Planning and Management, 130, pp. 4-14, 2004.
16. Chang, C. L., "A modified VIKOR method for multiple criteria analysis", Environmental Monitoring Assessment, 168, pp. 339-344, 2010.

17. Chang, C. L., and Hsu, C. H., "Applying a Modified VIKOR method to classify Land Subdivisions according to Watershed Vulnerability", Water Resource Management, 25, pp. 301-309, 2011.
18. Chang, C. L., and Hsu, C. H., "Multi-criteria analysis via the VIKOR method for prioritizing land-use restraint strategies in the Tseng-Wen reservoir watershed", International Journal of Environmental Management, Elsevier Science Publishers, 90, pp. 3226-3230, 2009.
19. Chen, L. Y., and Wang, T. C., "Optimizing partners' choice is IS/IT outsourcing projects: The strategic decision of fuzzy VIKOR", International Journal of Production Economics, Elsevier Science Publishers, 120, pp. 233-242, 2009.
20. Chen, S. J., and Hwang, C. L., "Fuzzy multiple attribute decision making: Methods and Applications", Springer-Verlag Publisher, Berlin, 1992.
21. Cristobal, S. J. R., "Multi-criteria decision-making in the selection of a renewable energy project in Spain: The Vikor method", Renewable Energy, Elsevier Science Publishers, 36, pp. 498-502, 2011.
22. Devi, K., "Extension of VIKOR method in intuitionistic fuzzy environment for robot selection", Expert Systems with Applications, Elsevier Science Publishers, 38, pp. 14163-14168, 2011.
23. Dubois, D., and Prade, H., "Operations on Fuzzy Numbers", International Journal of System Sciences, 9, pp. 613-626, 1978.
24. Duckstein, L., and Opricovic, S., "Multi objective Optimization in River Basin Development", Water Resources Research, 16(1), pp. 14-20, 1980.
25. Emovon, I., Norman, R. A., Murphy, A. J., and Pazouki, K., "An integrated multicriteria decision making methodology using compromise solution methods for prioritizing risk of marine machinery systems", Ocean Engineering, Elsevier Science Publishers, 105, pp. 92-103, 2015.
26. Escobar, M, T., and Moreno-Jimenez, J. M., "A linkage between the analytic hierarchy process and the compromise programming models", The International Journal of Management Science, Omega Publishers, 30, pp. 359-365, 2002.
27. Escobar, M. T., and Moreno-Jimenez, J. M., "The Hierarchical Compromise Programming", Sociedad de Estadistica e Investigacion Operativa Top, 5(2), pp. 253-281, 1997.
28. Fonseca, C. M., and Fleming, P. J., "Genetic Algorithms for Multiobjective Optimization: Formulation, Discussion and Generalization", Genetic Algorithms: proceedings of the Fifth International Conference (S. Forrest, ed.), San Mateo, California, 1993.
29. Hanss, M., "Applied Fuzzy Arithmetic: An Introduction with Engineering Applications", Springer-Verlag Berlin Heidelberg, ISBN 978-81-8489-300-7, 2005.
30. Hicks, J. R., "The Foundations of Welfare Economics", Economic Journal, 49, pp. 696-700, 1939.
31. Hippel, K. H., "Multiple objective decision making in water resources", Water Resources Bulletin, 28, pp. 3-10, 1992.
32. Hwang, C. L., Lin, M. J., "Group Decision Making under Multiple Criteria", Springer Publishers, Berlin, Germany, 1987.
33. Jahan, A., Mustapha, F., Ismail, M. Y., Sapuan, S. M., and Bahraminasab, M., "A comprehensive VIKOR method for material selection", Materials and Design, Elsevier Science Publishers, 32, pp. 1215-1221, 2011.
34. Kang, D., and Park, Y., "Review-based measurement of customer satisfaction in mobile service: Sentiment analysis and VIKOR approach", International Journal of Expert Systems with Applications, Elsevier Science Publishers, 41, pp. 1041-1050, 2014.
35. Kaya, T., and Kahraman, C., "Fuzzy multiple criteria forestry decision making based on an integrated VIKOR and AHP approach", Expert Systems with Applications, Elsevier Science Publishers, 38, pp. 7326-7333, 2011.
36. Keeney, R. L., and Raiffa, H., "Decisions with Multiple Objectives: Preferences and Value Tradeoffs", Cambridge University Press, Canada, 1993.
37. Keeney, R.L., Raiffa, H., and Rajala, D.W., "Decisions with multiple objectives: Preferences and value trade-offs", IEEE Transactions on Systems, Man and Cybernetics, SMC 9(7), pp. 403-403, 1979.
38. Klinger, A., "Vector-Valued Performance Criteria", IEEE Transactions on Automatic Control, 6, pp. 849-855, 1958.
39. Köksalan, M. M., Wallenius, J., and Zionts, S., "Multiple Criteria Decision Making: From Early History to the 21st Century", World Scientific, Singapore, 2011.
40. Koopmans, T. C., "Activity Analysis of Production and Allocation", Cowles Commission for Research in Economics, Monograph, John Wiley and Sons, New York, U.S.A, 1951.

41. Kuhn, H. W., and Tucker, A. W., "Nonlinear Programming", Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, university of California Press, Berkeley, California, pp. 39-54, 1951.
42. Kuo, M. S., and Liang, G. S., "Combining VIKOR with GRA techniques to evaluate servicer quality of airports under fuzzy environment", Expert Systems with Applications, Elsevier Science Publishers, 38, pp. 1304-1312, 2011.
43. Liao, H., and Xu, Z., "A VIKOR-based method for hesitant fuzzy multi-criteria decision making", 12, pp. 373-392, 2013.
44. Lin, J-H., Tzeng, G-H., and Jen, W., "Utilizing VIKOR to make ERP system supplier selection decision", Agriculture and Economics, 34, pp. 69-90, 2005.
45. Liu, H. C., Liu, L., and Wu, J., "Material selection using an interval 2-tuple linguistic VIKOR method considering subjective and objective weights", 52, pp. 158-167, 2013.
46. Liu, H. C., Mao, L. X., Zhang, Z. Y., and Li, P., "Induced aggregation operators in the VIKOR method and its application in material selection", International Journal of Applied Mathematical Modelling, Elsevier Science Publishers, 37, pp6325-6338, 2013.
47. Luc, D. T., "Pareto Optimality", In: Chinchuluun, A., Pardalos, P. M., Migdalas, A., and Pitsoulis, L., (eds) "Pareto Optimality, Game Theory and Equlibria", Springer Optimization and its Applications, Springer Science+Buisness Media, LLC, New York, U.S.A, 2008.
48. Mardani, A., Jusoh, A., and Zavadskas, E. K., "Fuzzy multiple criteria decision-making techniques and applications—Two decades review from 1994 to 2014". Expert Systems with Applications, 42, pp. 4126–4148, 2015.
49. Mardani, A., Jusoh, A., Md Nor, K., Khalifah, Z., Zakwan, N., and Valipour, A., "Multiple criteria decision-making techniques and their applications—A review of the literature from 2000 to 2014", Economic Research Ekonomska Istrazivanja, 28, pp. 516–571, 2015.
50. Mendel, J. M., and John, R. I. B., "Type-2 Fuzzy Sets Made Simple", IEEE Transactions on Fuzzy Systems, 10(2), pp. 117-127, 2002.
51. Miettinen, K., "Nonlinear Multiobjective Optimization", Springer Publishers, Berlin, Germany, 1999.
52. Miyamoto, S., "Multisets and Fuzzy Multisets", Soft Computing and Human-Centered Machines, Springer Publisher, pp. 9-33, 2000.
53. Mohammady, P., and Amid, A., "Integrated Fuzzy AHP and Fuzzy VIKOR Model for Supplier Selection in an Agile and Modular Virtual Enterprise", Fuzzy Information Engineering, 4, pp. 411-431, 2011.
54. Munasinghe, M., "Towards sustainable development: The Role of Enivronmental Economics and Valuation", The World Bank Environmental Paper Number 3, The World Bank, Washington, DC, U.S.A, 1992.
55. Musani, S., and Jemain, A. A., "Ranking Schools' Academic Performance Using a Fuzzy VIKOR", Journal of Physics: Conference Series, IOP Publishing, 622, pp. 1-10, 2015.
56. Olson, M., "The Logic of Collective Action: Public Goods and the Theory of Groups", Harvard Economic Studies, Vol. CXXIV, Harvard University Press, USA, 1965.
57. Opricovic, S., "Fuzzy VIKOR with an application to water resources planning", Expert Systems with Applications, Elsevier Science Publishers, 38, pp. 12983-12990, 2011.
58. Opricovic, S., and Djordjevic, B., "Optimal Long-Term Control of a multipurpose Reservoir with Indirect users", Water Resources Research, 12(6), pp. 1286-1290, 1976.
59. Opricovic, S., and Tzeng, G. H., "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS", European Journal of Operational Research, 156, pp. 445-455, 2004.
60. Opricovic, S., and Tzeng, G. H., "Extended VIKOR method in comparison with outranking methods", European Journal of Operational Research, Elsevier Science Publishers, 178, pp. 514-529, 2007.
61. Opricovic, S., Tzeng, G. H., "Multicriteria planning of post-earthquake sustainable reconstruction", Computer Aided Civil and Infrastructure Engineering, 17, pp. 211–220, 2002.
62. Park, J. H., Cho, H. J., and Kwun, Y. C., "Extension of the VIKOR method for group decision making with interval-valued intuitionistic fuzzy environment", Fuzzy Optimization Decision Making, 10, pp. 233-253, 2011.
63. Parnell, G. S., Bresnick, T. A., Tani, S. N., and Johnson, E. R., "Handbook of Decision Analysis", A John Wiley & Sons, Inc., Publication, Hoboken, New Jersey, 2013.
64. Pathinathan, T., and Ponnivalavan, K., "Pentagonal Fuzzy Numbers", International Journal of Computing Algorithm, 3, pp. 1003-1005, 2014.

65. Pathinathan, T., Ponnivalavan, K., and Dison, E. M., "Different Types of Fuzzy Numbers and Certain Properties", Journal of Computer and Mathematical Sciences, 6(11), pp. 631-651, 2015.
66. Quaddus, M. A., and Siddique, M. A. B., "Multiobjective Decision Support in Development Planning", Discussion Paper 94.01, ISSN 0811-6067, ISBN 0-86422-316-1.
67. Raj Kumar., and Pathinathan, T., "Sieving out the Poor using Fuzzy Decision Making Tools", Indian Journal of Science and Technology, 8(22), pp. 1-16, 2015.
68. Rajasekaran, K. G., Bhaskar, G. B., Murali, S., and Chandrasekaran, M., "An Emrcal Study on Selection of Supply Chain Partner by Multi-Criteria Decision Making Method VIKOR - A Case of Automotive Industry", International Journal of Engineering and Technology, 8(6), pp: 2997-3004, 2016.
69. Robbins, L., "An Essay on the Nature and Significance of Economic Science", 2nd Edition, Macmillan and Co Publishers, London, 1935.
70. Robbins, L., "Interpersonal Comparisons of Utility: A Comment", Economic Journal, 43, pp. 635-641, 1938.
71. Roy, B., "Problems and methods with multiple objective functions", Mathematical Programming, 1, pp. 239–266, 1971.
72. Saaty, T.L., "What is the Analytic Hierarchy Process?", Springer Publishers, Berlin, Germany, 1988.
73. Safari, H., Faraji, Z., and Majidian, S., "Identifying and evaluating enterprise aechitecture risks using FMEA and fuzzy VIKOR", Journal of Intelligent Manufacturing, Springer Publishers, New York, U.S.A, 2014.
74. Sanayei, A., Mousavi, S. F., and Yazdankhah, A., "Group decision making process for supplier selection with VIKOR under fuzzy environment", 37, pp. 24-30, 2010.
75. Sayadi, M. K., Heydari, M., and Shahanaghi, K., "Extension of VIKOR method for decision making problem with interval numbers", Applied Mathematical Modelling, Elsevier Science Publishers, 33, pp. 2257-2262, 2009.
76. Shemshadi, A., Shirazi, H., Toreihi, M., and Tarokh, M. J., "A fuzzy VIKOR method for supplier selection based on entropy measure for objective weighting", Expert Systems with Applications, Elsevier Science Publishers, 38, pp. 12160-12167, 2011.
77. Simonovic, S., "Decision Support Systems for Sustainable Management of Water Resources: General Principles", Water International, 21(4), pp. 223-232, 1996.
78. Srdjevic, B., Medeiros, Y. D. P., and Faria, A. S., "An Objective multi-criteria evaluation of water management scenarios", Water Resource Management, 18, pp. 35-54, 2004.
79. Srdjevic, B., Srdjevic, Zorica and Zoranovic, T., "PROMETHEE, TOPSIS and CP in Multicriteria Decision Making in Agriculture", 26(1), pp. 5-23, 2002.
80. Tiwari, D. N., "Sustainability criteria and cost-benefit analysis at the project level", In: Proceedings of International Conference on Integrating EIA and Cost-Benefit Analysis, University of Bradford, Bradford, UK, 1996.
81. Tiwari, D. N., Loof, R., and Paudyal, G. N., "Environmental-economic decision-making in lowland irrigated agriculture using multi-criteria analysis te4chniques", Agricultural Systems, Elsevier Science Publishers, 60, pp. 99-112, 1999.
82. Torra, V., "Hesitant Fuzzy Sets", International Journal of Intelligent Systems, 25(6), pp. 529-539, 2010.
83. Vahdani, B., Hadipour, H., Sadaghiani, J. S., and Amiri, M., "Extension of VIKOR method based on interval-valued fuzzy sets", International Journal of Advanced Manufacturing Technology, 47, pp. 1231-1239, 2010.
84. Wang, Y. L., and Tzeng, G. H., "Brand marketing for creating brand value based on a MCDM model combining DEMATEL with ANP and VIKOR methods", International Journal of Expert Systems with Applications, Elsevier Science Publishers, 39, pp. 5600-5615, 2012.
85. Yu, P. L., "A Class of solutions for group decision problems", Management Science, Vol. 19, No. 8, 1973.
86. Yu, P. L., "Multiple Criteria Decision Making: Concepts, Techniques and Extensions", Plenum Press, New York, U.S.A, 1985.
87. Yu, P.L., Cone Convexity, Cone Extreme, Points and Nondominated Solutions in Decision Problems with Multiobjectives, CSS 70-02, Center for System Science, Graduate School of Management, University of Rochester, 1972.
88. Zadeh, L. A., "Fuzzy Sets", Information and Control, 8, pp. 338-353, 1965.
89. Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning-I, Information Sciences, 8, pp. 199-249, 1975.

90. Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning-II, Information Sciences, 8, pp. 301-357, 1975.
91. Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning-III, Information Sciences, 8, pp. 43-80, 1975.
92. Zadeh, L. A., Optimality and Non Scalar-Valued Performance Criteria, IEEE Transactions on Automatic Control, 8, pp. 59-60, 1963.
93. Zandi, A., and Roghanian, E., "Extension of Fuzzy ELECTRE based on VIKOR", International Journal of Computers and Industrial Engineering, Elsevier Science Publishers, 66, pp. 258-263, 2013.
94. Zarghaami, M., "Integrated Water Resources Management in Polrud Irrigation System", Water Resources Management, Springer Publishers, 20, pp. 215-225, 2006.
95. Zeleny, M (Ed), "Multiple Criteria Decision Making Kyoto 1975", Lecture Notes in Economics and Mathematical Systems, Springer-Verlag Publishers, Heidelberg, 1976.
96. Zeleny, M., "An Essay into a philosophy of MCDM: A way of thinking or another algorithm?", Computers Ops Res, Pergamon Press Ltd, Great Britain, 19(7), pp. 563-566, 1992.
97. Zeleny, M., "Compromise Programming in Multiple Criteria Decision Making", edited by J. L. Cochrane and M. Zeleny, University of South Carolina Press, Columbia, pp. 262-301, 1973.
98. Zeleny, M., "Multiple Criteria Decision Making (MCDM): From Paradigm Lost to Paradigm Regained?" Journal of Multi-criteria Decision Analysis, 18, pp. 77-89, 2011.
99. Zeleny, M., "On the inadequacy of the regression paradigm used in the study of human judgment", Theory and Decision, D. Reidel Publishing Company, Dordrecht, Holland, 7, pp. 57-65, 1976.
100. Zhang, N., and Wei, G., "Extension of VIKOR method for decision making problem based on hesitant fuzzy set", International Journal of Applied Mathematical Modelling, Elsevier Science Publishers, 37, pp. 4938-4947, 2013.