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BALANCED DOUBLE LAYERED FUZZY GRAPH

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Abstract:

A development of Double Layered Fuzzy Graph (DLFG) is studied in this paper. From DLFG we construct Balanced Double Layered Fuzzy Graph (BDLFG) by modifying conditions. Some properties of BDLFG are derived and verified in this paper.

Key Words: Double layered fuzzy graph, complete fuzzy graph, balanced fuzzy graph, balanced double layered fuzzy graph.

1. Introduction:

A fuzzy graph has very vast applications in the field of computer science, social sciences, medical sciences etc. Zadeh in his paper fuzzy sets in 1965 introduced the concept of fuzzy relations which has a wide application in many uncertain real time problems such as pattern recognition and clustering [14]. Fuzzy graph theory was introduced by Azriel Rosenfeld in the year 1975 [15]. Yeh and Bang introduced various concepts in connectedness with fuzzy graphs [16]. Mordeson and Peng have contributed in the concept of operations on fuzzy graph [11]. Sunitha and Vijayakumar have developed the concept of complement of a fuzzy graph and also discussed about the operation union, join, cartesian product and composition on two fuzzy graphs [13]. Nagoorgani and K. Radha have studied the degree of vertex in some fuzzy graphs [8]. M.G. Karunambigai, M. Akram, S. Sivasankar and K. Palanivel have discussed on Balanced Intuitionistic Fuzzy Graphs [9]. T. Pathinathan and J. Jesintha Roseline introduced Double Layered Fuzzy Graph in the year 2014 [1] and developed matrix representation for DLFG in the same year [2].Further they characterized fuzzy graphs into different categories using arcs in fuzzy graphs [4]. In 2015, they developed structural core graph of DLFG [3]. In this paper we given a new approach to double layered fuzzy graph and introduce balanced double layered fuzzy graph further investigate different properties relating to BDLFG.

2. Preliminaries:

Definition 2.1 Let V be non-empty set. A fuzzy graph is a pair of function $G:(\sigma,\mu)$ where σ is a fuzzy subset of V, μ is a symmetric fuzzy relation on σ. i.e. $\sigma: V \to [0,1]$ and $\mu: V\times V \to [0,1]$ such that $\mu(\mathrm{u},\mathrm{v}) \le \sigma(\mathrm{u}) \wedge \sigma(\mathrm{v})$ for all u, v in V. The underlying crisp graph $G:(\sigma,\mu)$ is denoted as $G^*:(\sigma^*,\mu^*)$ where σ^* is referred to as the non-empty set V of vertices and $\mu^* = E \subseteq V\times V$.

Definition 2.2 A fuzzy graph $G:(\sigma,\mu)$ is complete if $\mu(\mathrm{u},\mathrm{v}) = \sigma(\mathrm{u}) \wedge \sigma(\mathrm{v})$ for all $u,v\in V$.

Definition 2.3 The density of a fuzzy graph $G:(\sigma,\mu)$ is $D(\mathrm{G}) = 2\left(\dfrac{\sum_{u,v\in\mu} \mu(\mathrm{u},\mathrm{v})}{\sum_{u,v\in\sigma} \sigma(\mathrm{u})\wedge\sigma(\mathrm{v})}\right)$.

Definition 2.4 A fuzzy graph $G:(\sigma,\mu)$ is balanced if $D(\mathrm{H}) \le D(\mathrm{G})$ for all fuzzy non-empty subgraphs H of G.

Example 2.4.1 Consider a fuzzy graph $G:(\sigma,\mu)$, such that

$\sigma = \{v_1,v_2,v_3,v_4\}$, $\mu = \{ (v_1,v_2), (v_2,v_3), (v_3,v_4), (v_4,v_1)\}$

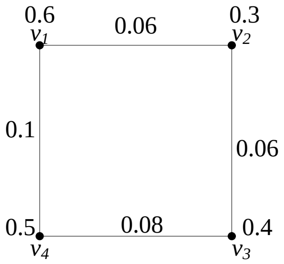

Figure 1: Fuzzy Graph

The density of the given graph is $D(G) = 2\left(\frac{0.06+0.06+0.08+0.1}{0.3+0.3+0.4+0.5}\right) = 2\left(\frac{0.3}{1.5}\right) = 0.4$.

Let $H_1 = (v_1, v_2)$, $H_2 = (v_2, v_3)$, $H_3 = (v_3, v_4)$, $H_4 = (v_4, v_1)$, $H_5 = (v_1, v_3)$, $H_6 = (v_1, v_2, v_3)$,
$H_7 = (v_1, v_3, v_4)$, $H_8 = (v_2, v_3, v_4)$, $H_9 = (v_1, v_2, v_3, v_4)$ be a non empty subgraphs of G.

The densities of the subgraphs are D (H_1) = 0.4, D (H_2) = 0.4, D (H_3) = 0.4, D (H_4) = 0.4, D (H_5) = 0, D (H_6) = 0.4, D (H_8) = 0.4, D (H_9) = 0.4.

So $D(H) \le D(G)$ for all subgraphs H of G. Hence G is balanced fuzzy graph.

3. Balanced Double Layered Fuzzy Graphs:

Let $G:(\sigma, \mu)$ be a fuzzy graph with the underlying crisp graph $G^*:(\sigma^*, \mu^*)$. The pair $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ is defined as follows. The vertex set of $BDL(G)$ be $\sigma^* \cup \mu^*$.

The fuzzy subset σ_{BDL} is define as

$\sigma_{BDL} = \sigma(u)$ if $u \in \sigma^*$

$\sigma_{BDL} = \mu(uv)$ if $uv \in \mu^*$

Case (i): The fuzzy relation μ_{BDL} on $\sigma^* \cup \mu^*$ is defined as

(i) $\mu_{BDL} = \mu(uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} = \mu(e_i) \wedge \mu(e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} = \sigma(u_i) \wedge \mu(e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise

Case (ii): The fuzzy relation μ_{BDL} on $\sigma^* \cup \mu^*$ is defined as

(i) $\mu_{BDL} = \mu(uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} = \mu(e_i) \wedge \mu(e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise

Case (iii): The fuzzy relation μ_{BDL} on $\sigma^* \cup \mu^*$ is defined as

(i) $\mu_{BDL} = \mu(uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} < \mu(e_i) \wedge \mu(e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} = \sigma(u_i) \wedge \mu(e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise

Case (iv): The fuzzy relation μ_{BDL} on $\sigma^* \cup \mu^*$ is defined as

(i) $\mu_{BDL} = \mu(uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} < \mu(e_i) \wedge \mu(e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise

4. Theoretical Concept on BDLFG:

Theorem 4.1:

Every complete balanced double layered fuzzy graph is balanced.

Proof:

Let $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a complete balanced double layered fuzzy graph, then by the definition we have,

(i) $\mu_{BDL} = \mu\ (uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} = \mu\ (e_i) \wedge \mu\ (e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} = \sigma\ (u_i) \wedge \mu\ (e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ Otherwise, for every $u, v \in V$.

Now $D(G) = 2\left(\dfrac{\sum_{u,v\in V} \mu(u,v)}{\sum_{u,v\in V} \sigma(u) \wedge \sigma(v)}\right)$, where $\sum_{u,v\in V} \mu\ (u,v) = \sum_{u,v\in V} \sigma\ (u) \wedge \sigma\ (v)$ we have $D(G) = 2$. i.e. the density of the every subgraphs of the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ will be equal to 2, i.e. $D(H_n) = 2, \Rightarrow D(H) = D(G)$. Hence the graph $G:(\mu_1, \mu_2)$ is a complete balanced double layered graph.

Example 4.1.2 Consider a double layered fuzzy graph $G:(\sigma, \mu)$ such that

$\sigma = \{v_1, v_2, v_3, e_1, e_2, e_3\}$,

$\mu = \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (e_1, e_2), (e_1, e_3), (e_2, e_3), (v_1, e_1), (v_2, e_2), (v_3, e_3)\}$.

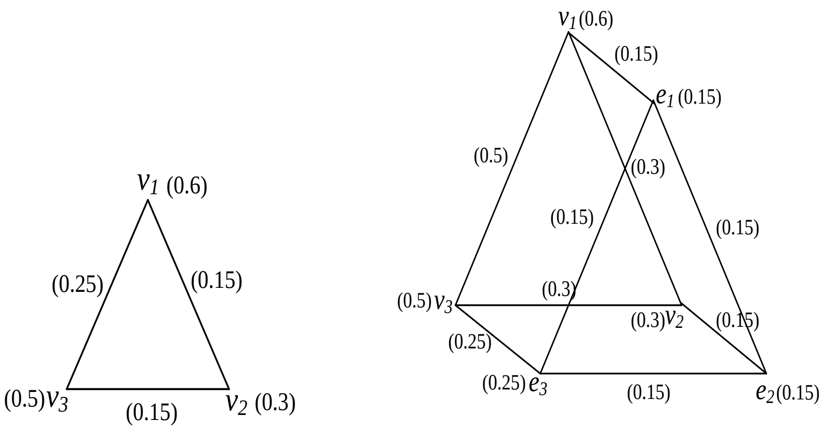

Figure 1.2: Complete Balanced Double Layered Fuzzy Graph

The density of given double layered fuzzy graph is $D(G) = 2\left(\dfrac{2.1}{2.1}\right) = 2$.

Let $H_1 = (v_1, v_2)$, $H_2 = (v_2, v_3)$, $H_3 = (v_3, v_1)$, $H_4 = (e_1, e_2)$, $H_5 = (e_1, e_3)$, $H_6 = (e_2, e_3)$, be a non empty subgraphs of G. The densities of the subgraphs are D (H_1) = 2, D (H_2) = 2, D (H_3) = 2, D (H_4) = 2, D (H_5) = 2, D (H_6) = 2. Shows that the density of subgraphs , H_4, H_5, H_6 are equal to the density of the graph G, i.e. $D(H) = D(G)$. Hence the graph is completely balanced.

Theorem 4.2:

Let $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a balanced double layered fuzzy graph, then by the definition, if $\mu_{BDL} = \mu\ (e_i) \wedge \mu\ (e_j)$ and $\mu_{BDL} < \sigma\ (u_i) \wedge \mu\ (e_i)$ then the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ is not balanced.

Proof:

Given $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a balanced double layered fuzzy graph, then by the definition of balanced double layered fuzzy graph

(i) $\mu_{BDL} = \mu\ (uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} = \mu\ (e_i) \wedge \mu\ (e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} < \sigma\ (u_i) \wedge \mu\ (e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise.

Now $D(G) = 2\left(\frac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right)$ and D (G) < 2, Since $\mu_{BDL} = \mu\ (e_i) \wedge \mu\ (e_j)$ the density of this subgraph will be equal to 2. *i.e.* $D(H_n) = 2$,

$\Rightarrow$ D(H) > D(G)

Hence case (ii) is not balanced.

Example 4.2.1 Consider a double layered fuzzy graph $G:(\sigma,\mu)$ such that

$\sigma = \{v_1, v_2, v_3, e_1, e_2, e_3\}$,

$\mu = \{ (u_1,u_2), (u_1,u_4), (u_2,u_3), (u_3,u_4), (v_1,v_2), (v_1,v_4), (v_2, v_3), (v_3, v_4), (u_1,v_1), (u_2,v_2), (u_3,v_3), (u_4,v_4)\}$.

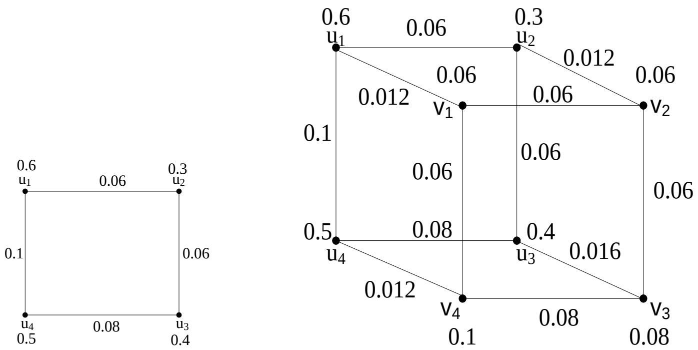

Figure 1.3: Unbalanced Double Layered Fuzzy Graph (i)

The density of given double layered fuzzy graph is $D(G) = 2\left(\frac{0.612}{2.06}\right) = 0.5941$.

Let $H_1 = (u_1,u_2)$, $H_2 = (u_2,u_3)$, $H_3 = (u_3,u_1)$, $H_4 = (v_1,v_2)$, $H_5 = (v_2,v_3)$, $H_6 = (v_3,v_4)$, be a non empty subgraphs of G.

The densities of the subgraphs are D (H_1) = 0.4, D (H_2) = 0.4, D (H_3) = 0.4, D (H_4) = 2, D (H_5) = 2, D (H_6) = 2. This shows that the density of subgraphs, H_4, H_5, H_6 are greater than the density of the graph G, i.e. D(H) > D(G). Hence the graph is not balanced.

Theorem 4.3:

Let $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a balanced double layered fuzzy graph, then by the definition, if $\mu_{BDL} < \mu\ (e_i) \wedge \mu\ (e_j)$ and $\mu_{BDL} = \sigma\ (u_i) \wedge \mu\ (e_i)$ then the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ is not balanced.

Proof:

Given $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a balanced double layered fuzzy graph, then by the definition of balanced double layered fuzzy graph

(i) $\mu_{BDL} = \mu\ (uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} < \mu\ (e_i) \wedge \mu\ (e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} = \sigma\ (u_i) \wedge \mu\ (e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise

Now $D(G) = 2\left(\frac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right)$ and D (G) < 2, Since $\mu_{BDL} = \sigma\ (u_i) \wedge \mu\ (e_i)$ then the density of this subgraph will be equal to 2, i.e. $\sum_{u,v\in V} \mu(u,v) = \sum_{u,v\in V} \sigma(u) \wedge \sigma(v)$,

$$i.e.\ D(H_n) = 2\frac{\sum_{u,v\in V} \mu(u,v)}{\sum_{u,v\in V} \sigma(u) \wedge \sigma(v)},$$

$$D(H_n) = 2\left(\frac{\sum_{u,v\in V} \mu(u,v)}{\sum_{u,v\in V} \mu(u,v)}\right) = 2,$$

$$\Rightarrow D(H) > D(G)$$

Hence the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ is not balanced.

Example 4.3.1 Consider a double layered fuzzy graph $G:(\mu_1, \mu_2)$ such that $\mu_1 = \{v_1, v_2, v_3, e_1, e_2, e_3\}$, $\mu_2 = \{(v_1,v_2), (v_1,v_3), (v_2, v_3), (e_1, e_2), (e_1, e_3), (e_2, e_3), (v_1,e_1), (v_2,e_2), (v_3,e_3)\}$.

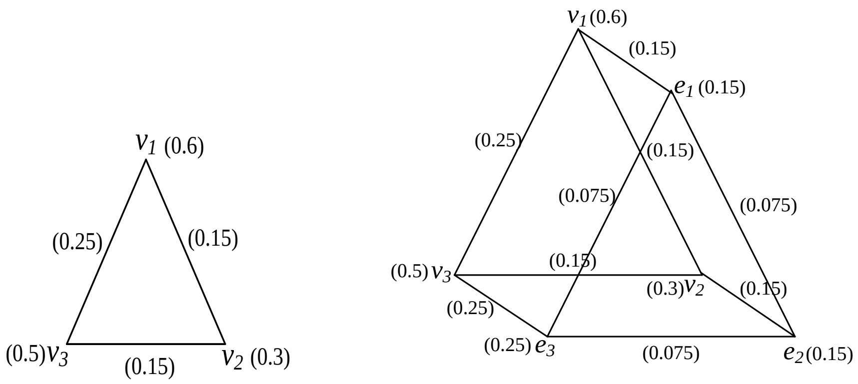

Figure 1.4: Unbalanced Double Layered Fuzzy Graph (ii)

The density of given double layered fuzzy graph is $D(G) = 2\left(\frac{1.325}{2.1}\right) = 1.261$.

Let $H_1 = (v_1, v_2)$, $H_2 = (v_2, v_3)$, $H_3 = (v_3, v_1)$, $H_4 = (v_1, e_1)$, $H_5 = (v_2, e_2)$, $H_6 = (v_3, e_3)$, be a non empty subgraphs of G. The densities of the subgraphs are D (H_1) = 1, D (H_2) = 1, D (H_3) = 1, D (H_4) = 2, D (H_5) = 2, D (H_6) = 2. This shows that the density of subgraphs, H_4, H_5, H_6 are greater than the density of the graph G, i.e. $D(H) > D(G)$. Hence the graph is not balanced.

Theorem 4.4:

Let $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a balanced double layered fuzzy graph, then by the definition, if $\mu_{BDL} < \mu(e_i) \wedge \mu(e_j)$ and $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$ then the graph $G:(\mu_1, \mu_2)$ is balanced.

Proof:

Given $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ be a balanced double layered fuzzy graph, then by the definition of balanced double layered fuzzy graph

(i) $\mu_{BDL} = \mu(uv)$ if $u \in \sigma^*$

(ii) $\mu_{BDL} < \mu(e_i) \wedge \mu(e_j)$ if the edge e_i and e_j have a node in common between them

(iii) $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$ if $u_i \in \sigma^*$ and $e_i \in \mu^*$ and each e_i is incident with single u_i either clockwise or anticlockwise

(iv) $\mu_{BDL} = 0$ otherwise

Now $D(G) = 2\left(\dfrac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right)$ and the density of the given graph will be D (G) < 2,

Since $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$

Then the density of this subgraph will be less than 2,

i.e. $\sum_{u,v\in V}\mu(u,v) < \sum_{u,v\in V}\sigma(u)\wedge\sigma(v)$

$$\therefore\ D(H_n) = 2\left(\frac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right) < 2.$$

$$\Rightarrow D(H) \le D(G)$$

Hence the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ is balanced.

Definition 4.5 Every balanced double layered complete graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ is strictly balanced if for every $u, v \in V$, $D(H) = D(G)$ for all non-empty subgraphs H of G.

Example: 4.6

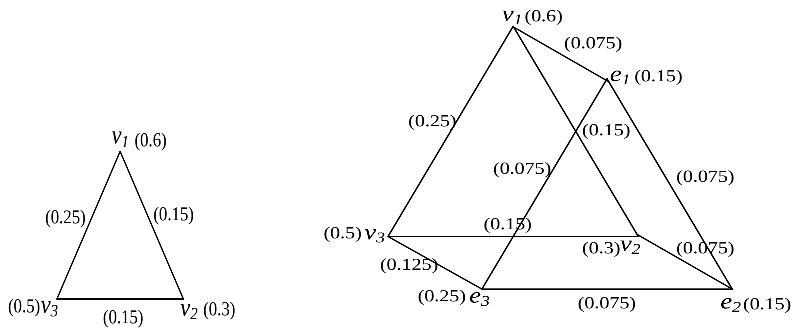

Figure 1.5: Balanced Double Layered Fuzzy Graph

The density of the given graph is calculated using the formulae

$$D(G) = 2\left(\frac{\sum_{u,v\in V}\mu(u,v)}{\sum_{u,v\in V}\sigma(u)\wedge\sigma(v)}\right).$$

$$D(G) = 2\left(\frac{0.15+0.15+0.25+0.075+0.075+0.075+0.075+0.075+0.125}{0.3+0.3+0.5+0.15+0.15+0.15+0.15+0.15+0.25}\right)$$

$$= 2\left(\frac{1.05}{2.1}\right) = 2(0.5) = 1.$$

The density of the subgraphs of the given graph is calculated using the same formulae and the values are listed in the following table (Table no.1), where we get $D(H) \le D(G)$.This shows that the given double layered fuzzy graph satisfies the condition $\mu_{BDL} < \mu(e_i) \wedge \mu(e_j)$, $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$ is balanced. Consider the sub graphs H_1 and H_{11}, we get $D(H_1) = \{v_1, v_2\} = 1$, and for H_{11} we get $D(H_{11}) = \{v_1, e_3\} = 0$.

Table 1: The densities of the subgraphs of the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ are listed.

S.No	Subgraphs of G(H_n)	D_{BDL}(H)	S.No	Subgraphs of G(H_n)	D_{BDL}(H)
1.	$H_1 = \{v_1v_2\}$	1	26.	$H_{26} = \{v_3e_3e_1\}$	1
2.	$H_2 = \{v_2v_3\}$	1	27.	$H_{27} = \{v_3e_3e_2\}$	1
3.	$H_3 = \{v_3v_1\}$	1	28.	$H_{28} = \{v_1v_2v_3e_3\}$	1
4.	$H_4 = \{e_1e_2\}$	1	29.	$H_{29} = \{v_2v_1v_3e_3\}$	1

5.	$H_5 = \{e_2 e_3\}$	1	30.	$H_{30} = \{v_2 v_3 v_1 e_1\}$	1
6.	$H_6 = \{e_3 e_1\}$	1	31.	$H_{31} = \{v_1 v_3 v_2 e_2\}$	1
7.	$H_7 = \{v_1 e_1\}$	1	32.	$H_{32} = \{v_3 v_1 v_2 e_2\}$	1
8.	$H_8 = \{v_2 e_2\}$	1	33.	$H_{33} = \{v_3 v_2 v_1 e_1\}$	1
9.	$H_9 = \{v_3 e_3\}$	1	34.	$H_{34} = \{e_1 e_2 e_3 v_3\}$	1
10.	$H_{10} = \{v_1 e_2\}$	0	35.	$H_{35} = \{e_3 e_2 e_1 v_1\}$	1
11.	$H_{11} = \{v_1 e_3\}$	0	36.	$H_{36} = \{e_1 e_3 e_2 v_2\}$	1
12.	$H_{12} = \{v_2 e_1\}$	0	37.	$H_{37} = \{e_2 e_1 e_3 v_3\}$	1
13.	$H_{13} = \{v_2 e_3\}$	0	38.	$H_{38} = \{v_1 e_1 e_3 v_3\}$	1
14.	$H_{14} = \{v_3 e_1\}$	0	39.	$H_{39} = \{v_1 e_1 e_2 v_2\}$	1
15.	$H_{15} = \{v_3 e_2\}$	0	40.	$H_{40} = \{v_2 e_2 e_1 v_1\}$	1
16.	$H_{16} = \{v_1 v_2 v_3\}$	1	41.	$H_{41} = \{v_2 e_2 e_3 v_3\}$	1
17.	$H_{17} = \{v_2 v_3 v_1\}$	1	42.	$H_{42} = \{v_1 e_1 e_3 e_3\}$	1
18.	$H_{18} = \{v_3 v_1 v_2\}$	1	43.	$H_{43} = \{v_1 e_1 e_3 e_2\}$	1
19.	$H_{19} = \{e_1 e_2 e_3\}$	1	44.	$H_{44} = \{v_2 e_2 e_1 e_3\}$	1
20.	$H_{20} = \{e_2 e_3 e_1\}$	1	45.	$H_{45} = \{v_2 e_2 e_3 e_1\}$	1
21.	$H_{21} = \{e_3 e_1 e_2\}$	1	46.	$H_{46} = \{v_3 e_3 e_2 e_1\}$	1
22.	$H_{22} = \{v_1 e_1 e_2\}$	1	47.	$H_{47} = \{v_3 e_3 e_1 e_2\}$	1
23.	$H_{23} = \{v_1 e_3 e_1\}$	1	48.	$H_{48} = \{e_3 e_1 e_2 v_2\}$	1
24.	$H_{24} = \{v_2 e_2 e_1\}$	1	49.	$H_{49} = \{e_1 v_1 v_3 e_3\}$	1
25.	$H_{25} = \{v_2 e_2 e_3\}$	1	50.	$H_{50} = \{e_1 v_1 v_2 e_2\}$	1

S.No	Subgraphs of G(H_n)	D_{BDL}(H)	S.No	Subgraphs of G(H_n)	D_{BDL}(H)
51.	$H_{51} = \{e_2 v_2 v_3 e_3\}$	1	79.	$H_{79} = \{v_3 e_3 e_1 e_2 e_3\}$	1
52.	$H_{52} = \{e_2 v_2 v_1 e_1\}$	1	80.	$H_{80} = \{e_3 e_2 e_1 v_1 e_3\}$	1
53.	$H_{53} = \{e_1 v_1 v_2 v_3\}$	1	81.	$H_{81} = \{e_2 e_1 e_3 v_3 e_1\}$	1
54.	$H_{54} = \{e_2 v_2 v_3 v_1\}$	1	82.	$H_{82} = \{e_1 e_3 e_2 v_2 e_1\}$	1
55.	$H_{55} = \{e_3 v_3 v_2 v_1\}$	1	83.	$H_{83} = \{e_2 e_1 e_3 v_3 e_2\}$	1
56.	$H_{56} = \{v_1 v_3 v_2 v_1\}$	1	84.	$H_{84} = \{e_1 v_1 v_3 e_3 e_1\}$	1
57.	$H_{57} = \{v_2 e_2 e_1 e_3 v_3\}$	1	85.	$H_{85} = \{e_1 v_1 v_2 e_2 e_1\}$	1
58.	$H_{58} = \{v_2 e_2 e_{.3} e_1 v_1\}$	1	86.	$H_{86} = \{e_2 v_2 v_3 e_3 e_2\}$	1
59.	$H_{57} = \{v_2 e_2 e_1 e_3 v_3\}$	1	87.	$H_{87} = \{e_2 v_2 v_1 e_1 e_2\}$	1
60.	$H_{60} = \{v_2 e_2 e_3 e_1 v_1\}$	1	88.	$H_{88} = \{e_1 v_1 v_2 v_3 e_1\}$	1
61.	$H_{61} = \{v_3 e_3 e_1 e_2 v_2\}$	1	89.	$H_{89} = \{e_2 v_2 v_3 v_1 e_2\}$	1
62.	$H_{62} = \{v_3 v_1 v_2 e_2 v_3\}$	1	90.	$H_{90} = \{e_3 v_3 v_2 v_1 e_3\}$	1
63.	$H_{63} = \{v_3 v_2 v_1 e_1 v_3\}$	1	91.	$H_{91} = \{e_1 v_1 v_2 v_3 e_3\}$	1
64.	$H_{64} = \{v_2 v_1 v_3 v_2 e_2\}$	1	92.	$H_{92} = \{e_1 v_1 v_3 v_2 e_2\}$	1
65.	$H_{64} = \{v_1 v_2 v_3 e_3 v_1\}$	1	93.	$H_{93} = \{e_2 v_2 v_1 v_3 e_3\}$	1

66.	$H_{66} = \{v_2 v_3 v_1 e_1 v_2\}$	1	94.	$H_{94} = \{e_2 v_2 v_3 v_1 e_1\}$	1
67.	$H_{67} = \{v_1 v_3 v_2 e_2 v_1\}$	1	95.	$H_{95} = \{e_3 v_3 v_2 v_1 e_1\}$	1
68.	$H_{68} = \{v_2 v_1 v_3 e_3 v_2\}$	1	96.	$H_{96} = \{e_3 v_3 v_1 v_2 e_2\}$	1
69.	$H_{69} = \{v_1 e_1 e_2 e_3 v_3\}$	1	97.	$H_{97} = \{v_1 e_1 e_3 e_2 v_2 v_3\}$	1
70.	$H_{70} = \{v_1 e_1 e_3 e_2 v_2\}$	1	98.	$H_{98} = \{v_2 e_1 e_1 e_3 v_3 v_1\}$	1
71.	$H_{71} = \{v_3 e_3 e_2 e_1 v_1\}$	1	99.	$H_{99} = \{v_2 e_2 e_3 e_1 v_1 v_3\}$	1
72.	$H_{72} = \{v_1 e_1 e_3 v_3 v_1\}$	1	100.	$H_{100} = \{v_3 e_2 e_2 e_1 v_1 v_2\}$	1
73.	$H_{73} = \{v_1 e_1 e_3 v_3 v_1\}$	1	101.	$H_{101} = \{v_3 e_3 e_1 e_2 v_2 v_1\}$	1
74.	$H_{74} = \{v_1 e_1 e_3 v_3 v_1\}$	1	102.	$H_{102} = \{e_2 v_2 v_3 v_1 e_1 e_2\}$	1
75.	$H_{75} = \{v_1 e_1 e_2 v_2 v_1\}$	1	103.	$H_{103} = \{e_3 v_3 v_2 v_1 e_1 e_3\}$	1
76.	$H_{76} = \{v_1 e_2 e_3 v_3 v_2\}$	1	104.	$H_{104} = \{e_1 v_1 v_2 v_3 e_3 e_1\}$	1
77.	$H_{77} = \{v_1 e_1 e_3 e_2 e_1\}$	1	105.	$H_{105} = \{e_1 v_1 v_3 v_2 e_2 e_1\}$	1
78.	$H_{78} = \{v_1 e_2 e_1 e_3 e_2\}$	1	106.	$H_{106} = \{e_2 v_2 v_1 v_3 e_3 e_2\}$	1

Example 4.7:

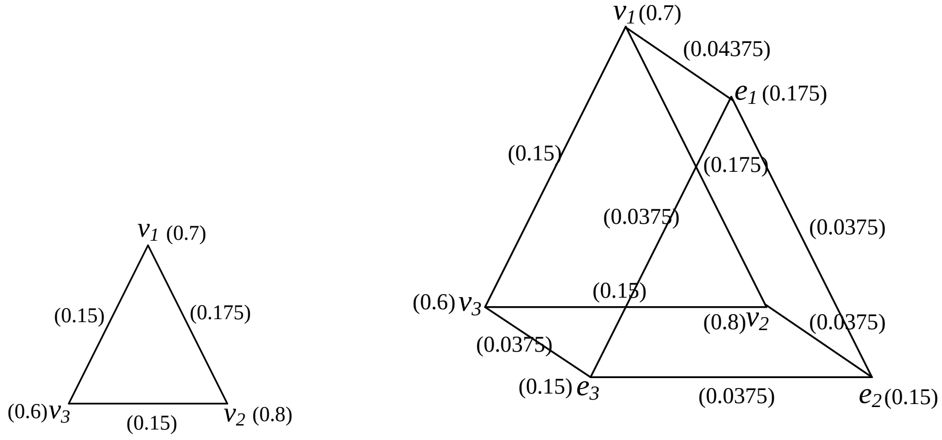

Figure 1.6: Balanced Double Layered Graph with density 0.5

The density of the given graph is calculated using the formulae

$$D(G) = 2\left(\frac{\sum_{u,v\in V} \mu(u,v)}{\sum_{u,v\in V} \sigma(u)\wedge\sigma(v)}\right).$$

$$\text{i.e. } D(G) = 2\left(\frac{0.70625}{2.825}\right) = 2(0.25) = 0.5$$

The density of the subgraphs of the given graph is attained using the same formulae and the values are listed in the following table (Table no. 2), were we get $D(H) = D(G) = 1$, which shows that the given double layered fuzzy graph to the condition $\mu_{BDL} < \mu(e_i) \wedge \mu(e_j)$, $\mu_{BDL} < \sigma(u_i) \wedge \mu(e_i)$ is balanced.

Table 2: The densities of the subgraphs of the graph $BDL(G):(\sigma_{BDL}, \mu_{BDL})$ are listed.

S.No	Subgraphs of G(H$_n$)	D$_{BDL}$(H)	S.No	Subgraphs of G(H$_n$)	D$_{BDL}$(H)
1.	$H_1 = \{v_1 v_2\}$	0.5	5.	$H_5 = \{e_2 e_3\}$	0.5
2.	$H_2 = \{v_2 v_3\}$	0.5	6.	$H_6 = \{e_3 e_1\}$	0.5
3.	$H_3 = \{v_3 v_1\}$	0.5	7.	$H_7 = \{v_1 e_1\}$	0.5
4.	$H_4 = \{e_1 e_2\}$	0.5	8.	$H_8 = \{v_2 e_2\}$	0.5

S.No	Subgraphs of G(Hn)	DBDL(H)	S.No	Subgraphs of G(Hn)	DBDL(H)
9.	$H_9 = \{v_3 e_3\}$	0.5	41.	$H_{41} = \{v_2 e_2 e_3 v_3\}$	0.5
10.	$H_{10} = \{v_1 e_2\}$	0	42.	$H_{42} = \{v_1 e_1 e_3 e_3\}$	0.5
11.	$H_{11} = \{v_1 e_3\}$	0	43.	$H_{43} = \{v_1 e_1 e_3 e_2\}$	0.5
12.	$H_{12} = \{v_2 e_1\}$	0	44.	$H_{44} = \{v_2 e_2 e_1 e_3\}$	0.5
13.	$H_{13} = \{v_2 e_3\}$	0	45.	$H_{45} = \{v_2 e_2 e_3 e_1\}$	0.5
14.	$H_{14} = \{v_3 e_1\}$	0	46.	$H_{46} = \{v_3 e_3 e_2 e_1\}$	0.5
15.	$H_{15} = \{v_3 e_2\}$	0	47.	$H_{47} = \{v_3 e_3 e_1 e_2\}$	0.5
16.	$H_{16} = \{v_1 v_2 v_3\}$	0.5	48.	$H_{48} = \{e_3 e_1 e_2 v_2\}$	0.5
17.	$H_{17} = \{v_2 v_3 v_1\}$	0.5	49.	$H_{49} = \{e_1 v_1 v_3 e_3\}$	0.5
18.	$H_{18} = \{v_3 v_1 v_2\}$	0.5	50.	$H_{50} = \{e_1 v_1 v_2 e_2\}$	0.5
19.	$H_{19} = \{e_1 e_2 e_3\}$	0.5	51.	$H_{51} = \{e_2 v_2 v_3 e_3\}$	0.5
20.	$H_{20} = \{e_2 e_3 e_1\}$	0.5	52.	$H_{52} = \{e_2 v_2 v_1 e_1\}$	0.5
21.	$H_{21} = \{e_3 e_1 e_2\}$	0.5	53.	$H_{53} = \{e_1 v_1 v_2 v_3\}$	0.5
22.	$H_{22} = \{v_1 e_1 e_2\}$	0.5	54.	$H_{54} = \{e_2 v_2 v_3 v_1\}$	0.5
23.	$H_{23} = \{v_1 e_3 e_1\}$	0.5	55.	$H_{55} = \{e_3 v_3 v_2 v_1\}$	0.5
24.	$H_{24} = \{v_2 e_2 e_1\}$	0.5	56.	$H_{56} = \{v_1 v_3 v_2 v_1\}$	0.5
25.	$H_{25} = \{v_2 e_2 e_3\}$	0.5	57.	$H_{57} = \{v_2 e_2 e_1 e_3 v_3\}$	0.5
26.	$H_{26} = \{v_3 e_3 e_1\}$	0.5	58.	$H_{58} = \{v_2 e_2 e_3 e_1 v_1\}$	0.5
27.	$H_{27} = \{v_3 e_3 e_2\}$	0.5	59.	$H_{57} = \{v_2 e_2 e_1 e_3 v_3\}$	0.5
28.	$H_{28} = \{v_1 v_2 v_3 e_3\}$	0.5	60.	$H_{60} = \{v_2 e_2 e_3 e_1 v_1\}$	0.5
29.	$H_{29} = \{v_2 v_1 v_3 e_3\}$	0.5	61.	$H_{61} = \{v_3 e_3 e_1 e_2 v_2\}$	0.5
30.	$H_{30} = \{v_2 v_3 v_1 e_1\}$	0.5	62.	$H_{62} = \{v_3 v_1 v_2 e_2 v_3\}$	0.5
31.	$H_{31} = \{v_1 v_3 v_2 e_2\}$	0.5	63.	$H_{63} = \{v_3 v_2 v_1 e_1 v_3\}$	0.5
32.	$H_{32} = \{v_3 v_1 v_2 e_2\}$	0.5	64.	$H_{64} = \{v_2 v_1 v_3 v_2 e_2\}$	0.5
33.	$H_{33} = \{v_3 v_2 v_1 e_1\}$	0.5	65.	$H_{64} = \{v_1 v_2 v_3 e_3 v_1\}$	0.5
34.	$H_{34} = \{e_1 e_2 e_3 v_3\}$	0.5	66.	$H_{66} = \{v_2 v_3 v_1 e_1 v_2\}$	0.5
35.	$H_{35} = \{e_3 e_2 e_1 v_1\}$	0.5	67.	$H_{67} = \{v_1 v_3 v_2 e_2 v_1\}$	0.5
36.	$H_{36} = \{e_1 e_3 e_2 v_2\}$	0.5	68.	$H_{68} = \{v_2 v_1 v_3 e_3 v_2\}$	0.5
37.	$H_{37} = \{e_2 e_1 e_3 v_3\}$	0.5	69.	$H_{69} = \{v_1 e_1 e_2 e_3 v_3\}$	0.5
38.	$H_{38} = \{v_1 e_1 e_3 v_3\}$	0.5	70.	$H_{70} = \{v_1 e_1 e_3 e_2 v_2\}$	0.5
39.	$H_{39} = \{v_1 e_1 e_2 v_2\}$	0.5	71.	$H_{71} = \{v_3 e_3 e_2 e_1 v_1\}$	0.5
40.	$H_{40} = \{v_2 e_2 e_1 v_1\}$	0.5	72.	$H_{72} = \{v_1 e_1 e_3 v_3 v_1\}$	0.5

S.No	Subgraphs of G(H_n)	D_{BDL}(H)	S.No	Subgraphs of G(H_n)	D_{BDL}(H)
73.	$H_{73} = \{v_1 e_1 e_3 v_3 v_1\}$	0.5	90.	$H_{90} = \{e_3 v_3 v_2 v_1 e_3\}$	0.5
74.	$H_{74} = \{v_1 e_1 e_3 v_3 v_1\}$	0.5	91.	$H_{91} = \{e_1 v_1 v_2 v_3 e_3\}$	0.5
75.	$H_{75} = \{v_1 e_1 e_2 v_2 v_1\}$	0.5	92.	$H_{92} = \{e_1 v_1 v_3 v_2 e_2\}$	0.5

76.	$H_{76} = \{v_1e_2e_3v_3v_2\}$	0.5	93.	$H_{93} = \{e_2v_2v_1v_3e_3\}$	0.5
77.	$H_{77} = \{v_1e_1e_3e_2e_1\}$	0.5	94.	$H_{94} = \{e_2v_2v_3v_1e_1\}$	0.5
78.	$H_{78} = \{v_1e_2e_1e_3e_2\}$	0.5	95.	$H_{95} = \{e_3v_3v_2v_1e_1\}$	0.5
79.	$H_{79} = \{v_3e_3e_1e_2e_3\}$	0.5	96.	$H_{96} = \{e_3v_3v_1v_2e_2\}$	0.5
80.	$H_{80} = \{e_3e_2e_1v_1e_3\}$	0.5	97.	$H_{97} = \{v_1e_1e_3e_2v_2v_3\}$	0.5
81.	$H_{81} = \{e_2e_1e_3v_3e_1\}$	0.5	98.	$H_{98} = \{v_2e_1e_1e_3v_3v_1\}$	0.5
82.	$H_{82} = \{e_1e_3e_2v_2e_1\}$	0.5	99.	$H_{99} = \{v_2e_2e_3e_1v_1v_3\}$	0.5
83.	$H_{83} = \{e_2e_1e_3v_3e_2\}$	0.5	100.	$H_{100} = \{v_3e_2e_2e_1v_1v_2\}$	0.5
84.	$H_{84} = \{e_1v_1v_3e_3e_1\}$	0.5	101.	$H_{101} = \{v_3e_3e_1e_2v_2v_1\}$	0.5
85.	$H_{85} = \{e_1v_1v_2e_2e_1\}$	0.5	102.	$H_{102} = \{e_2v_2v_3v_1e_1e_2\}$	0.5
86.	$H_{86} = \{e_2v_2v_3e_3e_2\}$	0.5	103.	$H_{103} = \{e_3v_3v_2v_1e_1e_3\}$	0.5
87.	$H_{87} = \{e_2v_2v_1e_1e_2\}$	0.5	104.	$H_{104} = \{e_1v_1v_2v_3e_3e_1\}$	0.5
88.	$H_{88} = \{e_1v_1v_2v_3e_1\}$	0.5	105.	$H_{105} = \{e_1v_1v_3v_2e_2e_1\}$	0.5
89.	$H_{89} = \{e_2v_2v_3v_1e_2\}$	0.5	106.	$H_{106} = \{e_2v_2v_1v_3e_3e_2\}$	0.5

5. Conclusion:

In this paper we have introduced the balanced double layered graph and given the conditions to make a double layered fuzzy graph to be balanced. Illustrated some examples to justify our definitions and results.

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