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ON CO-NORMAL PRODUCT OF TWO FUZZY GRAPHS

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Abstract:

New fuzzy graphs can be obtained from two given fuzzy graphs using different types of fuzzy graph products. In this paper, we define co-normal fuzzy graph product and determine the degree of vertices to the obtained fuzzy graph.

Key Words: Fuzzy Graph Products & Co-Normal Product of Fuzzy Graphs

1. Introduction:

Fuzzy graph is a useful tool used to analyze problems of combinatorial nature that arise in computer science, operations research, physical sciences, and economics and also to a number of real world situations such as chemical structure of a molecule, distribution of a product or service routes, communication lines, etc. Azriel Rosenfeld introduced Fuzzy graph theory in 1975 [9]. Mordeson and Peng have contributed on operations of union, join, cartesian products and composition on two fuzzy graphs [6]. Sunita and Vijayakumar have discussed various concepts in connectedness with fuzzy graph [11]. Nagoorgani and Radha have studied the degree of a vertex in some fuzzy graphs [1]. Radha and Arumugam have done their works on maximal products, lexico graphic products of two fuzzy graphs and studied its properties [7] [9]. Shovan Dogra have contributed to different types of product of fuzzy graphs and determined the degree of vertices of those graphs [8]. T. Pathinathan and J. Jesintha Roseline have shown the cartesian product of double layered fuzzy graph [2]. In this paper we have define Co-normal products on two fuzzy graphs and determine the degree of vertices of these new fuzzy graphs.

2. Preliminaries:

Definition 2.1 A fuzzy subset of a non-empty set V is a mapping $\sigma : V \rightarrow [0,1]$ and a fuzzy relation μ on a fuzzy subset σ , is a fuzzy subset of $V \times V$.

Definition 2.2 Let $G^* : (V, E)$ be a graph, σ be a fuzzy subset of V and μ be a fuzzy subset of $V \times V$. We call $G : (\sigma, \mu)$ product fuzzy graph if $\mu(x, y) < \sigma(x)\sigma(y)$ for all x, y$\in V$.

Definition 2.3 A product fuzzy graph $G : (\sigma, \mu)$ with underlying graph $G^* : (V, E)$ is said to be strong if $\mu(x, y) = \sigma(x)\sigma(y)$ for all x, y$\in$ E.

Definition 2.4 In the co-normal product of crisp graphs G and H , (g_1, h_1) and (g_2, h_2) are adjacent if and only if either g_1, g_2 are adjacent in G , or h_1, h_2 are adjacent in H . Co-normal product of crisp graph is also called as disjunctive product. It is denoted by $G * H$.

3. Co-Normal Product in Fuzzy Graphs:

3.1 Definition: Let $G_1 : (\mu_1, \mu_2)$ and $G_2 : (\mu_1', \mu_2')$ be two fuzzy graphs with underlying vertex set V_1 and V_2 and edge sets E_1 and E_2 respectively. Then co-normal product of G_1 and G_2 is a pair of functions $G_1 * G_2 : (\mu_1 * \mu_1', \mu_2 * \mu_2')$, with underlying vertex set $V_1 \times V_2 = \{(u_1, v_1) : u_1 \in V_1, v_1 \in V_2\}$ and underlying edge set $E_1 \times E_2 = \{((u_1, v_1)(u_2, v_2)) : u_1 = u_2, v_1v_2 \in E_2 \text{ or } u_1u_2 \in E_1, v_1 = v_2\}$ with

$$(\mu_1 * \mu_1')(u_1, v_1) = \mu_1(u_1) \wedge \mu_1'(v_1)$$

$$\mu_2 \times \mu_2'((u_1, v_1)(u_2, v_2)) = \begin{cases} \mu_2(u_1, u_2) \wedge \mu_1'(v_1) \text{ if } u_1u_2 \in E_1 \text{ and } v_1 = v_2 \\ \mu_2'(v_1, v_2) \wedge \mu_1(u_1) \text{ if } v_1v_2 \in E_2 \text{ and } u_1 = u_2 \\ \mu_2(u_1, u_2) \wedge \mu_1'(v_1) \wedge \mu_1'(v_2) \text{ if } u_1u_2 \in E_1 \text{ and } v_1v_2 \notin E_2 \\ \mu_2'(v_1, v_2) \wedge \mu_1(u_1) \wedge \mu_1(u_2) \text{ if } v_1v_2 \in E_2 \text{ and } u_1u_2 \notin E_1 \\ \mu_2(u_1, u_2) \wedge \mu_2'(v_1, v_2) \text{ if } u_1u_2 \in E_1 \text{ and } v_1v_2 \in E_2 \end{cases}$$

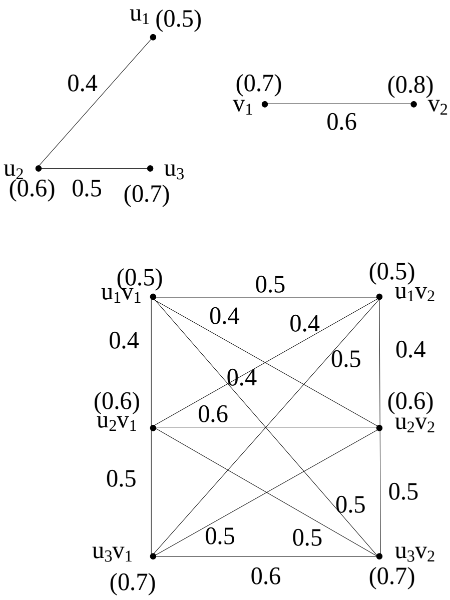


Figure 1: Co-normal product of two fuzzy graphs

Theorem 3.1:

The Co-normal product of two fuzzy graphs is a fuzzy graph.

Proof:

Let $G_1:(\mu_1,\mu_2)$ and $G_2:(\mu_1',\mu_2')$ denote two fuzzy graph with the underlying crisp graph $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ respectively. We want to prove that their Co-normal product $G_1 * G_2:(\mu_1*\mu_1',\mu_2*\mu_2')$ with underlying vertex set V_1*V_2 and edge set E_1*E_2 is again a fuzzy graph.

Let $((u_1,v_1)(u_2,v_2))\in E_1*E_2$. From the definition it follows that:

Case i) if $u_1u_2\in E_1$ and $v_1=v_2$

$$\begin{aligned}\mu_2\times\mu_2'((u_1,v_1)(u_2,v_2)) &= \mu_2(u_1,u_2)\wedge\mu_1'(v_1)\\ &\le (\mu_1(u_1)\wedge\mu_1(u_2))\wedge(\mu_1'(v_1)\wedge\mu_1'(v_2))\\ &= \mu_1(u_1)\wedge\mu_1'(v_1)\wedge\mu_1(u_2)\wedge\mu_1'(v_2)\\ &= (\mu_1*\mu_1')(u_1,v_1)\wedge(\mu_1*\mu_1')(u_2,v_2)\end{aligned}$$

Case ii) if $v_1v_2\in E_2$ and $u_1=u_2$

$$\begin{aligned}\mu_2\times\mu_2'((u_1,v_1)(u_2,v_2)) &= \mu_2'(v_1,v_2)\wedge\mu_1(u_1)\\ &\le(\mu_1'(v_1)\wedge\mu_1'(v_2))\wedge(\mu_1(u_1)\wedge\mu_1(u_2))\\ &= \mu_1'(v_1)\wedge\mu_1(u_1)\wedge\mu_1'(v_2)\wedge\mu_1(u_2)\\ &=(\mu_1*\mu_1')(u_1,v_1)\wedge(\mu_1*\mu_1')(u_2,v_2)\end{aligned}$$

Case iii) if $u_1,u_2\in E_1$ and $v_1,v_2\notin E_2$,

$$\begin{aligned}\mu_2\times\mu_2'((u_1,v_1)(u_2,v_2)) &= \mu_2(u_1,u_2)\wedge\mu_1'(v_1)\wedge\mu_1'(v_2)\\ &\le(\mu_1(u_1)\wedge\mu_1(u_2))\wedge\mu_1'(v_1)\wedge\mu_1'(v_2)\\ &=\mu_1(u_1)\wedge\mu_1'(v_1)\wedge\mu_1(u_2)\wedge\mu_1'(v_2)\\ &=(\mu_1*\mu_1')(u_1,v_1)\wedge(\mu_1*\mu_1')(u_2,v_2)\end{aligned}$$

Case iv) if $u_1,u_2\notin E_1$ and $v_1,v_2\in E_2$,

$$\mu_2 \times \mu_2'\left((u_1,v_1)(u_2,v_2)\right) = \mu_2'(v_1,v_2) \wedge \mu_1(u_1) \wedge \mu_1(u_2)$$
$$\leq (\mu_2'(v_1) \wedge \mu_2'(v_2)) \wedge \mu_1(u_1) \wedge \mu_1(u_2)$$
$$= \mu_2'(v_1) \wedge \mu_1(u_1) \wedge \mu_1(u_2) \wedge \mu_2'(v_2)$$
$$= (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Case v) if $u_1,u_2 \in E_1$ and $\mathrm{v}_1,\mathrm{v}_2 \in \mathrm{E}_2$

$$\mu_2 \times \mu_2'\left((u_1,v_1)(u_2,v_2)\right) = \mu_2(u_1,u_2) \wedge \mu_2'(v_1,v_2)$$
$$\leq (\mu_2(u_1) \wedge \mu_2(u_2)) \wedge \mu_2'(v_1) \wedge \mu_2'(v_2)$$
$$= (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Thus $\mu_2 \times \mu_2'((u_1,v_1)(u_2,v_2)) = (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$ shows that the co-normal product of two fuzzy graphs is a fuzzy graph.

Theorem 3.2:

The Co-normal product of two strong fuzzy graphs is a strong fuzzy graph.

Proof:

Let $G_1 : (\mu_1, \mu_2)$ and $G_2 : (\mu_1', \mu_2')$ denote two fuzzy graphs with the underlying crisp graph $G_1^* : (V_1, E_1)$ and $G_2^* : (V_2, E_2)$ respectively. We want to prove that their co-normal product of the two strong fuzzy graphs $G_1 * G_2 : (\mu_1 * \mu_1', \mu_2 * \mu_2')$ with underlying vertex set $V_1 \times V_2$ and edge set $E_1 \times E_2$ is again a strong fuzzy graph.

Let $\left((\mathrm{u}_1,\mathrm{v}_1)(\mathrm{u}_2,\mathrm{v}_2)\right) \in E_1 * E_2$.

From the definition it follows that:

Case i) if $u_1u_2 \in E_1$ and $v_1 = v_2$

$$\mu_2 \times \mu_2'\left((u_1,v_1)(u_2,v_2)\right) = \mu_2(u_1,u_2) \wedge \mu_1'(v_1)$$
$$= (\mu_1(u_1) \wedge \mu_1(u_2)) \wedge (\mu_1'(v_1) \wedge \mu_1'(v_2))$$ G_1 and G_2 are strong fuzzy graphs
$$= \mu_1(u_1) \wedge \mu_1'(v_1) \wedge \mu_1(u_2) \wedge \mu_1'(v_2)$$
$$= (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Case ii) if $v_1v_2 \in E_2$ and $u_1 = u_2$

$$\mu_2 \times \mu_2'\left((u_1,v_1)(u_2,v_2)\right) = \mu_2'(v_1,v_2) \wedge \mu_1(u_1)$$
$$= (\mu_1'(v_1) \wedge \mu_1'(v_2)) \wedge (\mu_1(u_1) \wedge \mu_1(u_2))$$ G_1 and G_2 are strong fuzzy graphs
$$= \mu_1'(v_1) \wedge \mu_1(u_1) \wedge \mu_1'(v_2) \wedge \mu_1(u_2)$$
$$= (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Case iii) if $u_1,u_2 \in E_1$ and $\mathrm{v}_1,\mathrm{v}_2 \notin \mathrm{E}_2$,

$$\mu_2 \times \mu_2'\left((u_1,v_1)(u_2,v_2)\right) = \mu_2(u_1,u_2) \wedge \mu_1'(v_1) \wedge \mu_1'(v_2)$$
$$= (\mu_1(u_1) \wedge \mu_1(u_2)) \wedge \mu_1'(v_1) \wedge \mu_1'(v_2)$$ G_1 and G_2 are strong fuzzy graph
$$= \mu_1(u_1) \wedge \mu_1'(v_1) \wedge \mu_1(u_2) \wedge \mu_1'(v_2)$$
$$= (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Case iv) if $u_1,u_2 \notin E_1$ and $\mathrm{v}_1,\mathrm{v}_2 \in \mathrm{E}_2$,

$$\mu_2 \times \mu_2'\left((u_1,v_1)(u_2,v_2)\right) = \mu_2'(v_1,v_2) \wedge \mu_1(u_1) \wedge \mu_1(u_2)$$
$$= (\mu_2'(v_1) \wedge \mu_2'(v_2)) \wedge \mu_1(u_1) \wedge \mu_1(u_2)$$ G_1 and G_2 are strong fuzzy graph
$$= \mu_2'(v_1) \wedge \mu_1(u_1) \wedge \mu_1(u_2) \wedge \mu_2'(v_2)$$
$$= (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Case v) if $u_1, u_2 \in E_1$ and $v_1, v_2 \in E_2$

$$\mu_2 \times \mu_2'((u_1,v_1)(u_2,v_2)) = \mu_2(u_1,u_2) \wedge \mu_2'(v_1,v_2)$$

$$=(\mu_2(u_1) \wedge \mu_2(u_2)) \wedge \mu_2'(v_1) \wedge \mu_2'(v_2) \text{ } G_1 \text{ and } G_2 \text{ are strong fuzzy graph}$$

$$=(\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$$

Thus $\mu_2 \times \mu_2'((u_1,v_1)(u_2,v_2)) = (\mu_1 * \mu_1')(u_1,v_1) \wedge (\mu_1 * \mu_1')(u_2,v_2)$ shows that the co-normal product of two strong fuzzy graphs is a strong fuzzy graph. Hence shows that the co-normal product of any two strong fuzzy graphs is again a strong fuzzy graph.

4. Degree of Vertex in Co-Normal Fuzzy Graph Product:

Let $G_1 : (\mu_1, \mu_2)$ and $G_2 : (\mu_1^*, \mu_2^*)$ be two fuzzy graphs with underlying vertex set V_1 and V_2 and edge sets E_1 and E_2 respectively. Then the degree of vertex (u_1, v_1),

Co-normal product in the fuzzy graph = $d_{G_1 * G_2}(u_1, v_1)$

$$= \sum_{u_1,u_2 \in E \text{ and } v_1,v_2 \in E} \mu_2(u_1,u_2) \wedge \mu_2'(v_1,v_2) + \sum_{u_1,u_2 \in E \text{ and } v_1,v_2 \notin E} \mu_2(u_1,u_2) \wedge \mu_1'(v_1) \wedge \mu_1'(v_2)$$

$$+ \sum_{u_1,u_2 \notin E \text{ and } v_1,v_2 \in E} \mu_1(u_1) \wedge \mu_1(u_2) \wedge \mu_2'(v_1,v_2) + \sum_{u_1,u_2 \in E \text{ and } v_1 = v_2} \mu_2(u_1,u_2) \wedge \mu_1'(v_1)$$

$$+ \sum_{v_1,v_2 \in E \text{ and } u_1 = u_2} \mu_2'(v_1,v_2) \wedge \mu_1(u_1).$$

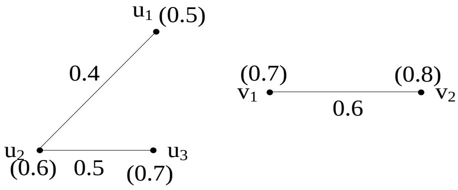
$$= \sum_{u_1,u_2 \in E \text{ and } v_1,v_2 \in E} \mu_1(u_1,u_2) \wedge \mu_2(v_1,v_2)$$, if both the fuzzy graphs G_1 and G_2 are complete.

Case i) if $\mu_2 \le \mu_2'$

$$d_{G_1 * G_2}(u_1, v_1) = \sum_{u_1,u_2 \in E} \mu_2(u_1,u_2) = d_{G_1}(u_1)$$

Case ii) if $\mu_2' \le \mu_2$

$$d_{G_1 * G_2}(u_1, v_1) = \sum_{v_1,v_2 \in E} \mu_2(v_1,v_2) = d_{G_2}(v_1).$$



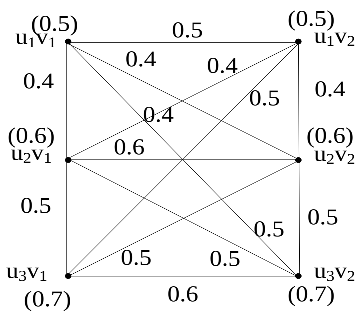


Figure 2: Co-normal product of two fuzzy graphs

Here $\mu_2 \le \mu_2'$.The degree of the vertex $d_{G_1 * G_2}(u_1, v_1)$ = (0.5+0.4+0.5+0.4) = 1.8 = $d_{G_1}(u_1)$.

5. Conclusion:

In this paper we have defined the co-normal product of two fuzzy graphs and we have determined the degree of the vertices. This co-normal product can also be extended to other types of fuzzy graphs such as intuitionistic fuzzy graph.

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