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COMPUTER REPRESENTATION OF GRAPHS USING BINARY LOGIC CODES IN DISCRETE MATHEMATICS

S. Geetha* & Dr. S. Jayakumar**
* Assistant Professor, Department of Mathematics, Bon Secours College for Women, Villar Bypass, Thanjavur, Tamilnadu
** Head, Department of Mathematics, Bon Secours College for Women, Villar Bypass, Thanjavur, Tamilnadu

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Abstract:

Discrete Mathematics is a fundamental component of mathematics and computer science. It is the study of finite systems. The Digital computer is basically a finite structure and many of its properties can be understood and interpreted within the framework of finite mathematical systems. Graph are represented by means of Diagrams. These Graphs may be considered as Graph of certain relation. Graphs, Directed Graphs appear in many areas of Mathematics and Computer Science. Graphs are defined as an abstract mathematical system. Elements of Graph Theory are indispensable in almost all areas of Computer Science. The use of a Digital Computer in solving graph theoretic problems is undoubtedly by an important part of learning Graph theory. A Pictorical representation of a Graph is very convenient for a visual study and so it is better for computer Processing. A Matrix is a convenient and useful way of representing a Graph to a Computer. A Binary Relation in a set V is defined as a subset of V x V. It was shown that such a relation could be represented atleast in some cases by a Diagram which are called the Graph of the relation. Given a suitable Digraph G=(V, E), it is necessary to assume some kind of ordering of the nodes of the graph in the sense that a particular node is called a first node another a second node and so on. The Matrix representation of Graph depends upon the ordering of the nodes. Let G = (V, E) be a simple digraph in which V ={v_1, v_2, v_3, …v_n} and these nodes are assumed to be ordered form v_1 to v_n. A Diagrammatic Representation of a Graph has limited usefulness. Such a representation is only possible when the number of nodes and edges is reasonably small. The Main result of Matrix algebra can be readily applied to study the Structural properties of graphs from an algebraic point of view. In many applications of graph theory such as in electrical network analysis, Computer Algorithms, Matrices also turn out to be the natural way of expressing the problem. In this paper there are most frequently used Matrix representations of a Graphs such as the Incidence matrix, Adjacency matrix, Path Matrix, Circuit matrix, were given. Also the correspondence between some graph theoretic properties and matrix properties will be given. A Matrix whose entries are the elements of a two element Boolean algebra, where B ={0, 1} is called a Boolean Matrix. Any Matrix whose elements are either 0 or 1 is called a bit matrix or Boolean Matrix. An n×n matrix whose elements are 0 and 1. An n×n matrix the Incidence Matrix, Adjacency Matrix, Path Matrix, Circuit Matrix contains 0 and 1. The concepts and notations of graphs in Discrete Mathematics are useful in describing objects and problem in branches of computer science, computer algorithms, programming, languages, cryptography, artificial automated theorem and software development. While a graph will be concluded as an abstract mathematical system in significant part of its several diagrams such as Computer representations and pictures of graphs must be mathematically verified.

Key Works: Graph, Digraph, Matrix, Binary Relation, Nodes, Edges, Boolean Matrix, Computer Algorithms, Incidence Matrix, Adjacency Matrix, Path Matrix & Circuit Matrix.

1. Introduction:

A graph is represented as an abstract Mathematical system. A Binary relation in a set V is defined as a subset of VxV. It was shown that such a relation could be represented atleast in some cases by a Diagram which are called the Graph of the relation. Given a suitable Digraph G =(V,E) it is necessary to assume some kind of ordering of the nodes of the graph in the sense that a particular node is called a first node another a second node and so on. The Matric representation of Graph depends upon the ordering of the nodes. Let G =(V, E) be a simple digraph in which V ={V_1, V_2, V_3 …..V_n} and these nodes are assumed to be ordered form V_1 to V_n. A diagrammatic Representation of a Graph has limited use fullness. Such a representation is only possible when the number of nodes and adges is reasonably small. The main results of Matrix algebra can be readily applied to study the structural properties graphs from an algebraic point of view.

In many application is of graph theory. Such as in electrical network analysis, computer algorithms. Matrices also turn out to be the natural way of expressing the problem. The concepts and notations of graphs in Discrete Mathematics, are useful in describing objects and problem in branches of computer science, computer algorithm, programming, Languages, Cryptography, artificial automated theorem and software development.

2. Basic Concepts of Graph Theory:

A graph G = ⟨V, E, ϕ⟩ consists of a nonempty set V called the set of nodes (points, vertices) of the graph, E is said to be the set of edges of the graph, and ϕ is a mapping from the setoff edges E to a set of ordered pairs of elements of V. In a graph G = ⟨V, E⟩ an edge which is associated with an ordered pair of Vx V is called a directed edge of G, while an edge which is associated with an unordered pair of nodes is called an undirected edge. A graph in which every edge is directed is called a digraph, or a directed graph. Let ⟨V, E⟩ be a graph and let x ∈ E be a directed edge associated with the ordered pair of nodes ⟨u, *v*⟩. Then the edge x is said to be initialing or originating in the node u and terminating or ending in the node *v*. The nodes u and *v* are also called the initial and terminal nodes of the edge x. An edge x ∈E which joins the nodes u and v, whether it be directed or undirected, is said to be incident to the nodes u and *v*. Any graph which contains some parallel edges is called a multigraph. On the other hand, if there is no more than one edge between a pair of nodes no more than one directed edge in the case of a directed graph, then such a graph is called a simple graph.

In a directed graph, for any node u the number of edge which have u as their initial node is called the out degree of the node v. The number of edges which have v as their terminal node is called the in degree of v, and the sum of the out degree and the in degree of a node v is called its total degree. In case of an undirected graph, the total degree or the degree of a node v is equal to the number of edges incident with v. Let V(H) be the set of nodes on a graph H and V(G) be the set of nodes of a graph G such that V(H) ⊆ V(G). If, in addition, every edge of H is also an edge of G, then the graph H is called a subgraph of the graph G, which is expressed by writing H⊆G. Let G = ⟨V, E⟩ be a simple digraph. Consider a sequence of edges of G such that the terminal node of any edge in the sequence is the initial node on the next edge, if any in the sequence. An example of such a sequence is

$$(\langle v_{i1,} v_{i2} \rangle,\ \langle v_{i2,} v_{i3} \rangle, \ldots.., \langle v_{i_{k-2},} v_{i_{k-1}} \rangle,\ \langle v_{ik-1,} v_{ik} \rangle)$$

Where it is assumed that all the nodes and edges appearing in the sequence are in V and E respectively. It is customary to write such a sequence as

$$(v_{i1,} v_{i2}, \ldots.., v_{i_{k-1},} v_{i_{k-1,}} v_{ik})$$

Any sequence of edges of a digraph such that the terminal node of any edge in the sequence is the initial node of the edge, if any, appearing next in the sequence defines a path of the graph. A path is said to traverse through the nodes appearing in the sequence, originating in the initial node of the first edge and ending in the terminal node of the last edge in the sequence. The number of edges appearing in the sequence of a path is called the length of the path. Consider the simple digraph given in Fig.

Some of the paths originating in node 1 and ending in node 3 are

P_1 = (⟨ 1, 2⟩, ⟨2, 3⟩)
P_2 = (⟨1, 4⟩, ⟨4, 3⟩)
P_3 = (⟨1, 2⟩, ⟨2, 4⟩, ⟨4, 3⟩)
P_4 = (⟨1, 2⟩, ⟨2, 4⟩, ⟨4, 1⟩, ⟨1, 2⟩, ⟨2, 3⟩)
P_5 = (⟨1, 2⟩, ⟨2, 4⟩, ⟨4, 1⟩, ⟨1, 4⟩, ⟨4, 3⟩)
P_6 = (⟨1, 1⟩, ⟨1, 1⟩, ……, ⟨1, 2⟩, ⟨2, 3⟩)

A path which originates and ends in the same node is called a cycle (circuit).

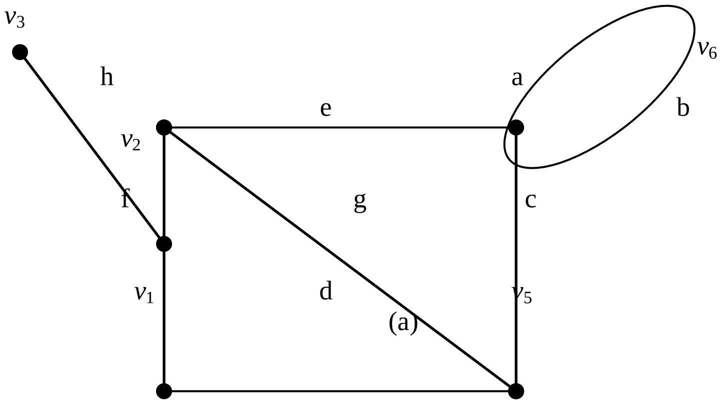
3. Matrix Representation of Graphs Using Binary Logic Codes:

A pictorial representation of a graph is very convenient for a visual study, other representations are better for computer processing. A matrix is a convenient and useful way of representing a graph to a computer. Any known results of matrix algebra can be readily applied to study the structural properties of graphs from an algebraic point of view. In many applications is of graph theory, such as in electrical network analysis and operations research, matrices also turn out to be the natural way of expressing the problem. Also a correspondence between some graph theoretic properties and matrix properties will be established. Let G be a graph with n vertices, e edges, and no self-loops. Define an n by e matrix A = [a_{ij}], whose n rows correspond to the n vertices and the e columns correspond to the e edges, as follows.

The matrix element

a_{ij} = 1, if jth edge e_j is incident on ith vertex v_i, and
= 0, otherwise.

Such a matrix A is called the vertex-edge incidence matrix, or simply incidence matrix. Matrix A for a graph G is sometimes also written as A(G). A graph and its incidence matrix are shown in Fig.(a)



	a	b	c	d	e	f	g	h
v_1	0	0	0	1	0	1	0	0
v_2	0	0	0	0	1	1	1	1
v_3	0	0	0	0	0	0	0	1
v_4	1	1	1	0	1	0	0	0
v_5	0	0	1	1	0	0	1	0
v_6	1	1	0	0	0	0	0	0

(b) The incidence matrix contains only two elements, 0 and 1. Such a matrix is called a binary matrix or a (0, 1)-matrix. These two elements are from Galois field modulo 2. Given any geometric representation of a graph without self-loops, we can readily write its incidence matrix. The following observation about the incidence matrix A can be made.

- ✓ Since very edge is incident on exactly two vertices, each column of A has exactly two 1's.
- ✓ The number of 1's in each row equals th degree of the corresponding vertex.
- ✓ A row with all 0's, therefore, represents an isolated vertex.
- ✓ If a graph G is disconnected and consists of two components g_1 and g_2, the incidence matrix A(G) of graph G can be written in a block diagonal from as

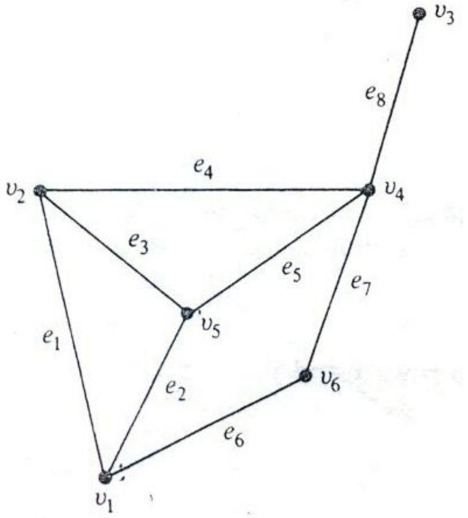
$$A(G) = \left[\begin{array}{c:c} A(g_1) & 0 \\ \hdashline 0 & A(g_2) \end{array}\right]$$

- ✓ Permutation of any two rows or columns in an incidence matrix simply corresponds to relabeling the vertices and edges of the same graph.

4. Adjacency Matrix:

The adjacency matrix of a graph G with n vertices and no parallel edges is an n by n symmetrix binary matrix X = [x_{ij}] defined over the ring of integers such that

x_{ij} =1, if there is an edge between ith and jth vertices, and



$$X = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

A simple graph and its adjacency matrix are shown in Fig. The following observations that can be made about the adjacency matrix X of a graph G are

- ✓ The entries along the principle diagonal of X are all 0's if and only if the graph has so self-loops. A self-loop at the ith vertex corresponds to x_{ii}=1.
- ✓ The definition of adjacency matrix makes no provision for parallel edges. This is why the adjacency matrix X was defined for graphs without parallel edges.
- ✓ If the graph has no self-loops (and no paerallel edges, of course), the degree of a vertex equals the number of 1's in the corresponding row or column of X.
- ✓ A graph G is discounnected and is in two components g_1 and g_2 if and only if its adjacency matrix X(G) can be partitioned as

$$X(G) = \left(\begin{array}{c|c} X(g_1) & 0 \\ \hline 0 & X(g_2) \end{array}\right)$$

Where $X(g_1)$ is the adjacency matrix of the component g_1 and $X(g_2)$ is that of the component g_2.

5. Path Matrix:

A path matrix is defined for a specific pair of vertices in a graph, say (x, y), and is written as P (x, y). The rows in P(x, y) correspond to different paths between vertices x and y, and the columns correspond to the edges in G. That is, the path matrix for (x, y) vertices is P (x, y) = [p_{ij}], where

$$p_{ij} \quad = 1, \quad \text{if ith edge lies in ith path, and}$$
$$\quad = 0, \quad \text{otherwise.}$$

Consider all path between vertices v_3 and v_4 in Fig. (a). There are three different paths; {h, e}, {h, g, c}, and {h, f, d, c}. Let us number them 1, 2, and 3, respectively. Then we get the 3 by 8 path matrix P (v_3, v_4)

$$P(v_{3,}v_4) = \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

Some of the observations one can make at once about a path matrix P(x, y) of a graph G are

- ✓ A column of all 0's corresponds to an edge that does not lie in any path between x and y.
- ✓ A column of all 1's corresponds to an edge that lies in every path between x and y.
- ✓ There is no row with all 0's.

6. Circuit Matrix:

Let the number of different circuits in a graph G be q and the number of edges in G be e. Then a circuit matrix B = [b_{ij}] of G is a q by e, (0, 1) matrix defined as follows.

$$b_{ij} \quad = 1, \quad \text{if ith circuit includes jth edge, and}$$
$$\quad = 0, \quad \text{Otherwise.}$$

The graph in Fig. (a) has four different circuits, {a, b}, {c, e, g}, {d, f, g}, and {c, d, f, e}. Therefore, its circuit matrix is a 4 by 8, (0, 1) –matrix as shown:

$$B(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{pmatrix} a & b & c & d & e & f & g & h \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The following observations can be made about a circuit matrix B(G) of a graph G.

- ✓ A column of all zeros corresponds to a noncircuit edge (i.e., an edge that does not belong to any circuit).
- ✓ Each row of B(G) is a circuit vector.

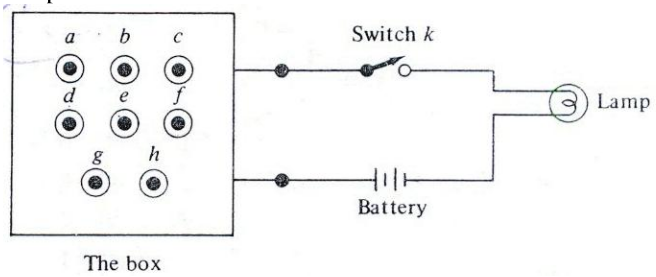
- ✓ The number of 1's in a row is equal to the number of edges in the corresponding circuit.
- ✓ If graph G is separable (or disconnected) and consists of two blocks (or components) g_1 and g_2, the circuit matrix B(G) can be written in a block-diagonal form as

$$B(G) = \left(\begin{array}{c|c} B(g_1) & 0 \\ \hline 0 & B(g_2) \end{array} \right)$$

- ✓ Permutation of any two rows or columns in a circuit matrix simply corresponds to relabeling the circuits and edges.

7. Application to a Switching Network:

Suppose that a box contains a switching network consisting of eight switches a, b, c, d, e, f, g, and h. The switches can be turned on or off from outside to determine how the switches are connected inside the box, without opening the box, of course. One way to find the answer is to connect a lamp at the available terminals in series with a battery and an additional switch k, as shows in Fig. And then find out which of the various combinations light up the lamp.



In this experiment, suppose you discover that the combinations that turn on the lamp are eight:

(a, b, f, h, k), (a, b, g, k), (a, e, f, g, k), (a, e, h, k),
(b, c, e, h, k), (c, f, h, k), (c, g, k), (d, k).

Solution:

Consider the switching network as a graph whose edges represent switches. We can assume that the graph is connected, and has no self loop. Since a lit lamp implies the formation of a circuit, we can regard the preceding list as a partial list of circuits in the corresponding graph. With this list we form a circuit matrix.

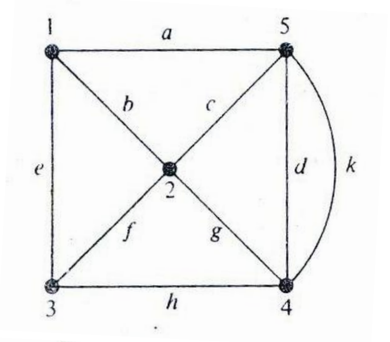
$$B = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccccccc} a & b & c & d & e & f & g & h & k \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

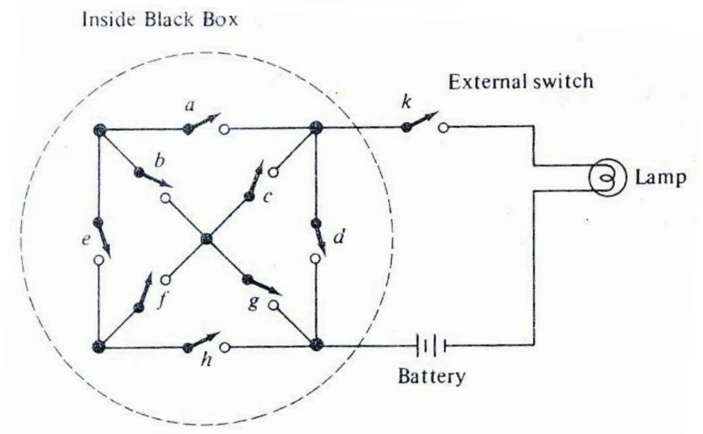
Next, to simplify the matrix, we should remove the obviously redundant circuits. Observe that the following ring sums of circuits give rise to other circuits.

$(a, b, g, k) \oplus (c, f, h, k) \oplus (c, g, k) = (a, b, f, h, k),$
$(a, b, g, k) \oplus (a, e, h, k) \oplus (c, g, k) = (b, c, e, h, k),$
$(a, e, h, k) \oplus (c, f, h, k) \oplus (c, g, k) = (a, e, f, g, k).$

$$A = \begin{array}{c} \begin{array}{ccccccccc} b & e & f & g & d & a & c & h & k \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \end{array}$$

From the incidence matrix A we can readily construct the graph and hence the corresponding switching network, as shown in Fig.





Graph and the corresponding switching network

8. Conclusion:

The theory of matrices has been brought to been upon the theory of graphs. Graph theory also plays an important role in several areas on computer science such as switching theory and logical design. Graph will be concluded as an abstract Mathematical system in significant part of its several diagrams such as computer representations and pictures of graphs are mathematically verified.

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