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FEA OF HEAT TRANSFER WITH HEAT SOURCE AND VISCOUS DISSIPATION UNDER CONVECTIVE BOUNDARY CONDITIONS

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Abstract:

Finite element analysis of heat transfer through Cu-EG nanofluid along a stretching sheet is presented in this paper. Variable thermal conductivity is considered and heat source is included in the energy equation. Viscous dissipation is also considered. The convective boundary conditions are imposed to solve the governing equations. The results are presented for velocity and temperature for various non- dimensional parameters graphically. The rate of heat transfer and shear stress are tabulated. Results are compared with the previous study.

Key Words: EG-Cu Nano-Fluid, Heat Source, Viscous Dissipation, Convective Boundary, Variable Thermal Conductivity & Stretching Sheet

Introduction:

The fast development of heat transfer using nano-fluid has been observed since Choi [1] invented nano fluid due to high thermal conductivity. Variety of nano fluids (particles like Al2O3, CuO etc... In the solvents Ethelene Glycol, water etc.) are widely used in industries like oil refineries etc. Recently, Rahman [3] has evaluated the MHD flow over a flat plate with partial slip subjected to the convective surface heat flux at the boundary. He reported that the local similarity solution should be applied due to the dependency of slip coefficient on *x*-coordinate.

Significant work on the dynamics of boundary layer flow over stretching surface was done by Crane [9], who examined the two-dimensional Navier-Stokes equations. Later on, various aspects of the problem have been investigated by Dutta et al. [10], and Chen and Char [11]. Khan and Pop [12] presented the boundary layer flow of nanofluid past a stretching sheet. Recently, Hassani et al. [13] studied an analytical solutionfor boundary layer flow of a nanofluid past a stretching sheet. Rana and Bhargava [14] analyzed the flow and heat transfer over a nonlinear stretching sheet, a numerical study. Hamad and Ferdows [15] studied the similarity solution of boundary layer stagnation-point flow towards a heated porous stretching sheet saturated with a nanofluid with heat absorption/generation and suction/blowing: a Lie group analysis. Recently Srikanth et.al. [4] Reported that the temperature reduces with increase in rate of stretching in Cu-Water nanofluid. Yao *et al*. [2] have investigated the convective boundary condition along a stretching/shrinking sheet.

The heat source/sink effects in thermal convection, are significant where there may exist a high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reactions. Vajravelu and Hadjinicalaou [17] have studied on hydrodynamic convective heat transfer from a stretching surface with heat generation/absorption. Molla et al. [18]

Studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation are studied by Samad et al. [19]. Viscous dissipation effects are usually ignored in macro scale systems, in laminar flow in particular, except for very viscous liquids at comparatively high velocities. However, even for common liquids at laminar Reynolds numbers, frictional effects in micro scale systems may change the energy equation [20]. Koo and Kleinstreuer [6] have investigated the effects of viscous dissipation on the temperature field using dimensional analysis and experimentally validated computer simulations. Three common working fluids—water, methanol and *iso*-propanol—in different conduit geometries have been considered in this study. The authors concluded that the channel size was a key factor that determines the impact of viscous dissipation. Furthermore, viscous dissipation effects may be very significant for fluids with high viscosities and low specific heat capacities, even at relatively low Reynolds numbers. Accordingly, the viscous dissipation term should be considered in the micro scale systems.

All the above literature, did not explain the heat transfer with heat source and viscous dissipation through EG-Cu nano-fluid. So these studies motivated to study the heat transfer of EG-Cu nano fluid along a stretching sheet with convective boundary conditions under temperature dependent heat source and viscous dissipation when the thermal conductivity is not constant. Similarity transformation method has used. The present paper is an extension of Bhaskar Reddy et.al.[5] with heat source and viscous dissipation using finite element method.

Mathematical Formulation:

A steady laminar Ethelene Glycol (EG) based copper (Cu) nano-fluid flow along a vertical permeable stretching sheet is assumed. The flow is due to the stretching of the sheet along x-axis. Keeping the origin fixed the vertical sheet is stretched with velocity $u_w(x)=ax$,where a is constant. The transverse magnetic field is applied. A schematics drawing with coordinate system is shown in Figure.1. The governing equations under the other regular assumptions are as follows.

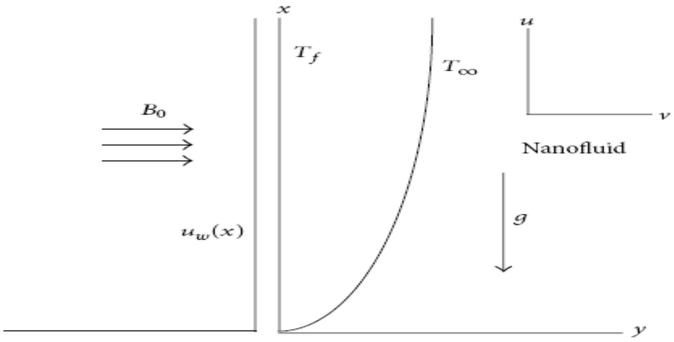

FIGURE 1: Physical model of the system.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}[\mu_{nf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{nf}(T-T_\infty)-\sigma B_0^2 u] \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{(\rho c_p)_{nf}}\frac{\partial}{\partial y}\left(k_{nf}^*\frac{\partial T}{\partial y}-q_r\right)+\frac{Q}{(\rho c_p)_{nf}}(T-T_\infty)+\frac{\mu_{nf}}{(\rho c_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

The boundary conditions are as follows

$$u=u_w+A\frac{\partial u}{\partial y},\quad v=-v_w,\quad -k_f\frac{\partial T}{\partial y}=h_f(T_f-T_\infty)\quad \text{at}\quad y=0$$

$$u\to 0,\quad T\to T_\infty\quad \text{as}\quad y\to\infty \qquad (4)$$

where u and V are the velocity components in the x and y directions, respectively, T is the temperature of the nanofluid, T_∞ is the ambient fluid temperature, g is the acceleration due to gravity, σ is the electric conductivity, B_0 is the uniform magnetic field strength, and q_r is the radiative heat flux. A is the velocity slip factor. *vw* is the wall mass flux with *vw* < 0 for suction and *vw* > 0 for injection, respectively. k_f is the thermal conductivity of the ordinary fluid and T_f is the temperature of the hot fluid.

The variable thermal conductivity expressed by Arunachalam and Rajappa [7], Chiam [8] and Seddeek and Salem [16] linearly as

$$k^*=k[1+\varepsilon\theta] \qquad (5)$$

Where k is thermal conductivity parameter.

The following physical quantities for nano-fluid established by various researchers are considered,

$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$, $(\rho c_p)_{nf}=(1-\phi)(\rho c_p)_f+\phi(\rho c_p)_s$,
$(\rho\beta)_{nf}=(1-\phi)(\rho\beta)_f+\phi(\rho\beta)_s$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad \frac{k_{nf}}{k_f} = \frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)} \tag{6}$$

The sub scripts nf, f and s represents nano-fluid, fluid and metal particle respectively. The thermo physical properties are given in the following table.

Table 1: Thermo Physical Properties

Physical Properties	Ethylene-Glycol	Cu
Cp (J/kgK)	1114.4	385
ρ (kg/m3)	2415	8933
$\beta \times 10^{-5}$ (1/K)	65	1.67
k (W/m K)	0.252	401

By using Rosseland diffusion approximation, $q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$ (7)

Where σ^* is the Stefan-Boltzman constant and k^* is the mean absorption coefficient. The fluid is assumed to be optically thick for which the above model holds good. The temperature differences are assumed to be sufficiently small so that T^4 can be expressed using Taylor series around T_∞ after neglecting higher order terms as follows,

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

The equations (1) – (3) are non dimensionalised using the following similarity transformation,

$$\eta = \left(\frac{a}{\nu_f}\right)^{1/2} y, \; u = a x f'(\eta), \; v = -(a\nu_f)^{1/2}, \; \theta = \frac{T-T_\infty}{T_f - T_\infty}, \; M = \frac{\sigma B_0^2}{a\rho_f}, \; \lambda = \frac{g(\rho\beta_f)(T_f-T_\infty)}{\rho_f a u_w}, \; Nr = \frac{4\sigma^* T_\infty^3}{k^* k_{nf}}$$

$$Pr = \frac{\nu_f}{\alpha_f}, \; S = -\frac{v_w}{\sqrt{a\nu_f}}, \; Q_H = \frac{Q}{a(\rho C_p)_f}, \; Ec = \frac{u_w^2}{(T_f-T_\infty)(C_p)_f}, \; \delta = A\sqrt{\frac{a}{\nu_f}}, \; \Gamma = \frac{h_f}{k_f}\sqrt{\frac{\nu_f}{a}} \tag{9}$$

Where ν_f is the kinematic viscosity.

In view of equations (1) – (9), the governing equations are reduced as follows,

$$f''' + (1-\phi)\left[\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left(f f'' - f'^2\right) + \left(1-\phi+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right)\lambda\theta - M f'\right] = 0 \tag{10}$$

$$\left(1+\frac{4}{3}Nr\right)\theta'' + \varepsilon\,\theta\,\theta'' + \varepsilon\theta'^2 + \Pr\left(\frac{k_f}{k_{nf}}\right)\left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right) f\theta' + \left(\frac{k_f}{k_{nf}}\right)Q_H\,\theta + \left(\frac{k_f}{k_{nf}}\right)Ec = 0 \tag{11}$$

Where 'denotes differentiation with respect to η. The corresponding boundary conditions are,

$$f'(0) = 1 + \delta f''(0), \quad f(0) = S, \quad \theta'(0) = -\Gamma(1-\theta(0)), \quad f'(\infty)\to 0, \; \theta(\infty)\to 0 \tag{12}$$

The local rate of heat transfer (Nusselt Number) and skin friction respectively in dimensionless form are defined as follows,

$$Nu_x = -\frac{1}{\mathrm{Re}^{1/2}}\frac{k_{nf}}{k_f}\theta'(0)$$

$$C_f = -\frac{1}{\mathrm{Re}^{1/2}}\frac{1}{(1-\phi)^{2.5}} f''(0)$$

Where $\mathrm{Re} = \frac{u_w x}{\nu_f}$ is the Reynolds Number.

Results and Discussion:

The novelty of the paper lies in solving the coupled non linear equations (10) & (11) subject to the boundary conditions (12) using Galerkin FEM as explained by J. N. Reddy. Initially η is limited to a auxiliary symbol for computational convenience. The one dimensional domain is divided into 100 parts for computation and the Galerkin finite element method is applied with help of Mathematica 10.4 package. The limit of η is adjusted till the satisfaction of boundaries achieved. During the application of FEM the iteration process terminated at 10^{-5} accuracy. The results are presented here for Cu-EG based nanofluid because literature reports that more impact is found in EG base nano fluid than water based nanofluid.. The results are validated by comparing with Bhaskar Reddy. N. et. al. [5].

The results are presented from Figs. 2 – 15. All the velocity and temperature profiles are clearly exhibiting the behavior of the flow and temperature for various parameters. From Fig.2 the momentum boundary layer thickness increases with the volume fraction from 0% to 20% of the solid particles in the fluid, this in turn enhances the thermal boundary layer thickness with solid volume fraction, it is observed from Fig.3. This is due to the high thermal conductivity of nanofluid. The increase in thermal conductivity increases the momentum and thermal boundary layers, observed from Figs.4 & 5. Figs. 6&7 exhibits the influence of radiation on momentum and temperature respectively. The profiles are spread clearly, indicating the significance of radiation over conduction. The influence of temperature dependent heat source on momentum and temperature are shown in Figs. 8 & 9. The increase in heat source enhances the momentum and thermal boundary layers. This happens due to the presence of Cu- particles. The profiles of viscous dissipation on momentum and temperature are shown in Figs. 10 & 11. The reduction of enthalpy enhances the momentum and temperature slightly. Figs. 12 & 13 show the influence of magnetic field on momentum and temperature respectively. The presence of magnetic field retards the flow thus enhances the temperature. The force resist the flow due to magnetic field is called Lorentz force, which dominates near the base of the plate. Fig.14. Fig.15 represents the convective parameter effect on temperature. The temperature increases with convection.

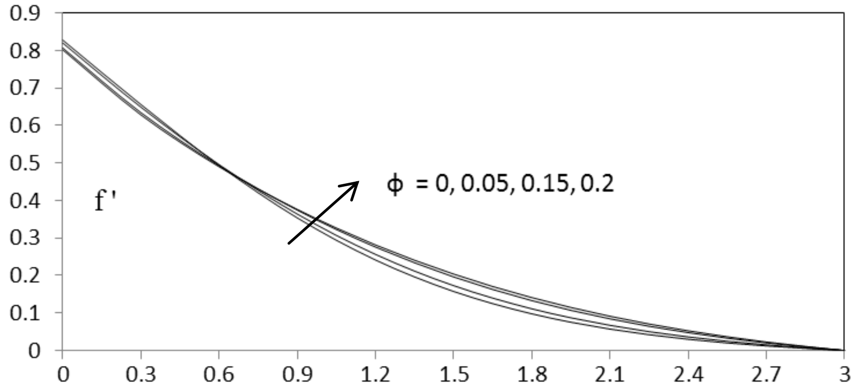

Figure 2: velocity profiles with φ.

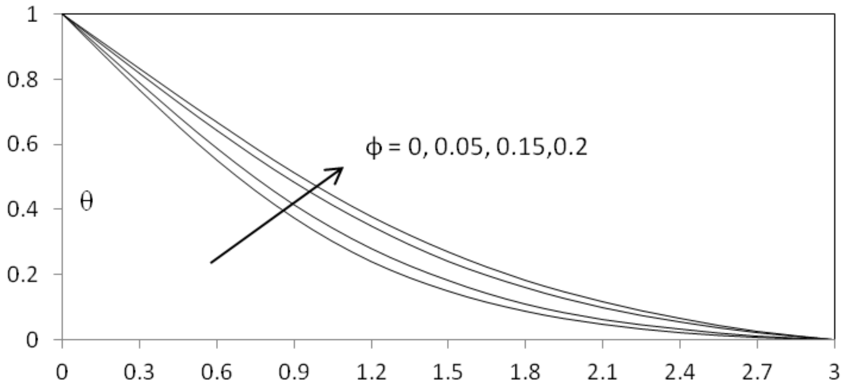

Figure 3: temperature profile with φ.

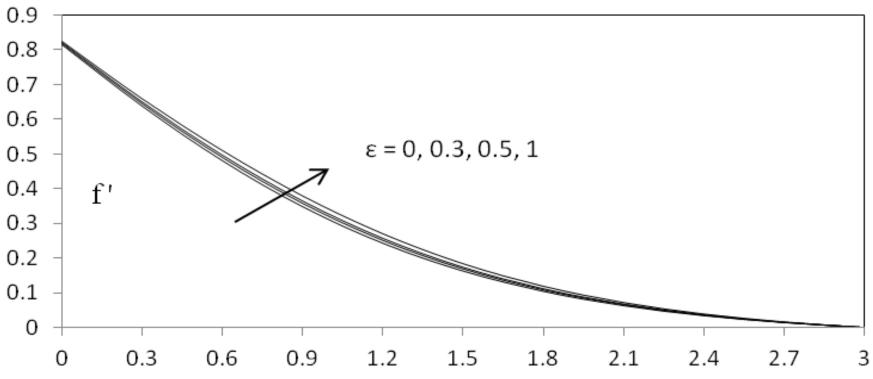

Figure 4: velocity profile with ε.

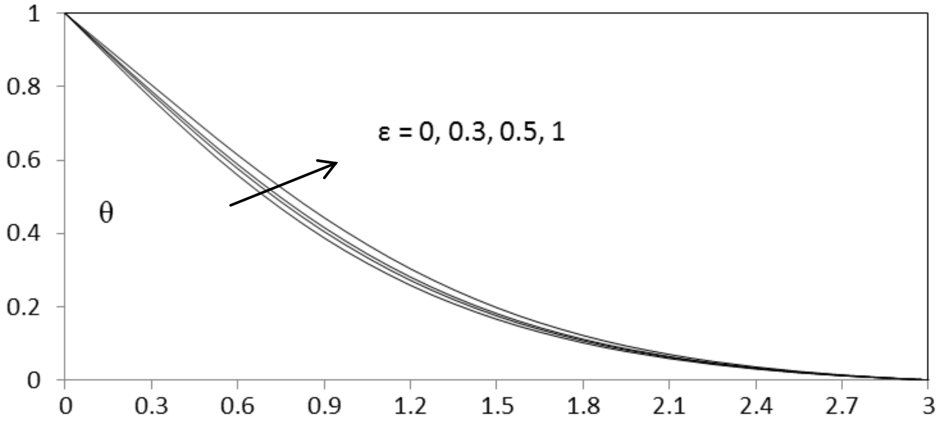

Figure 5: temperature profile with ε.

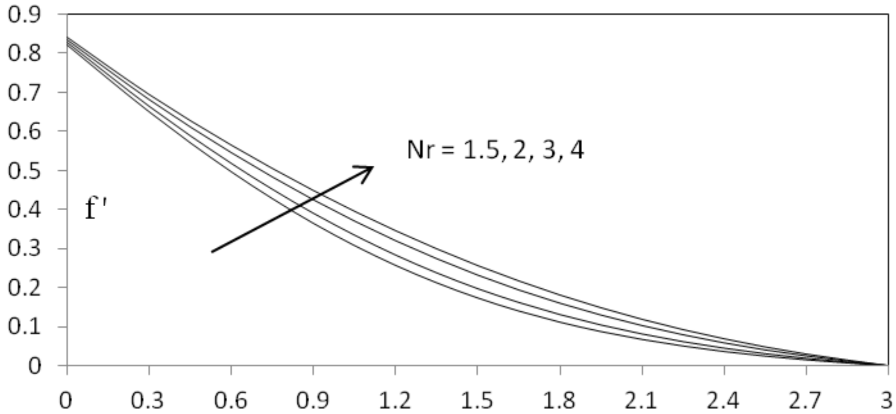

Figure 6: velocity profile with Nr.

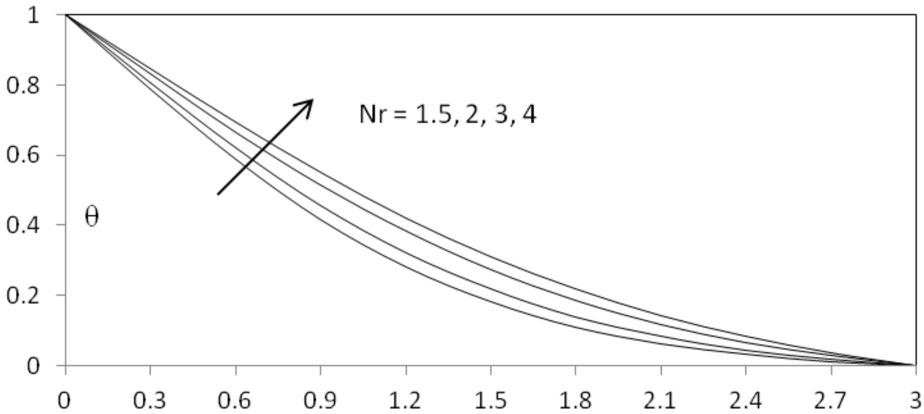

Figure 7: temperature profile with Nr.

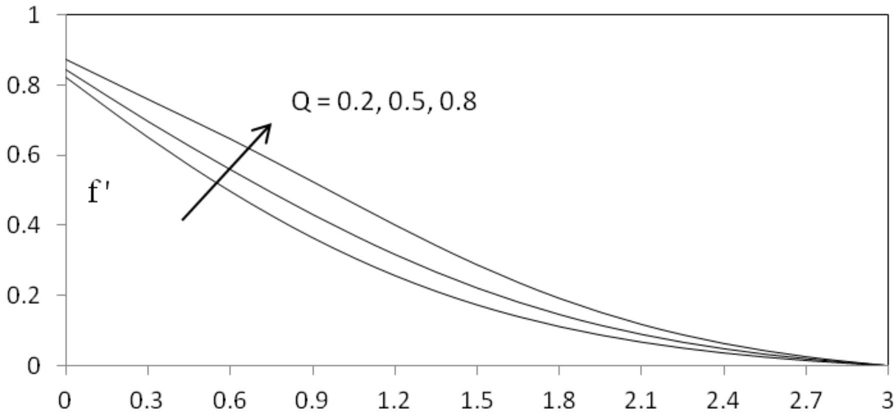

Figure 8: velocity profile with Q.

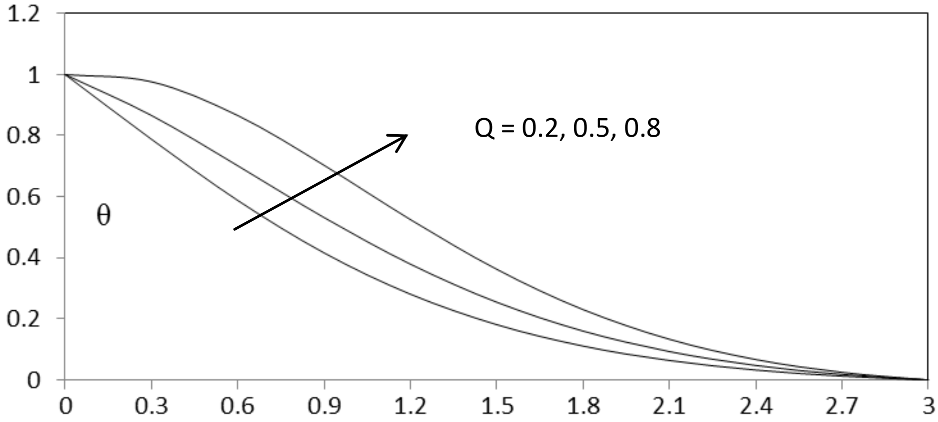

Figure 9: temperature profile with Q.

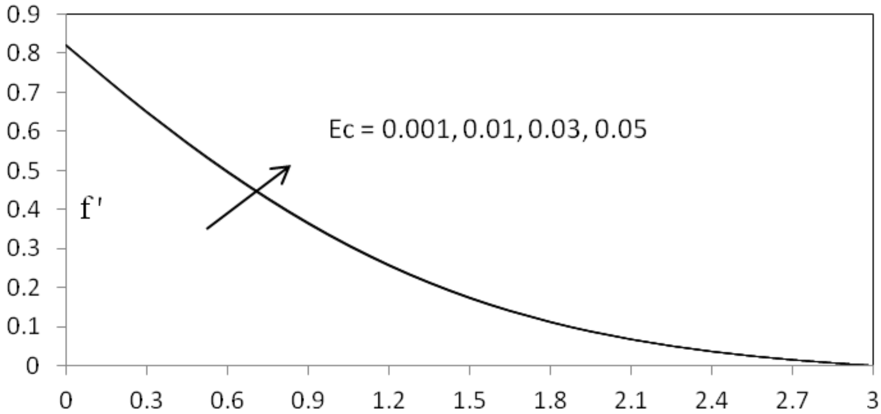

Figure 10: velocity profile with Ec.

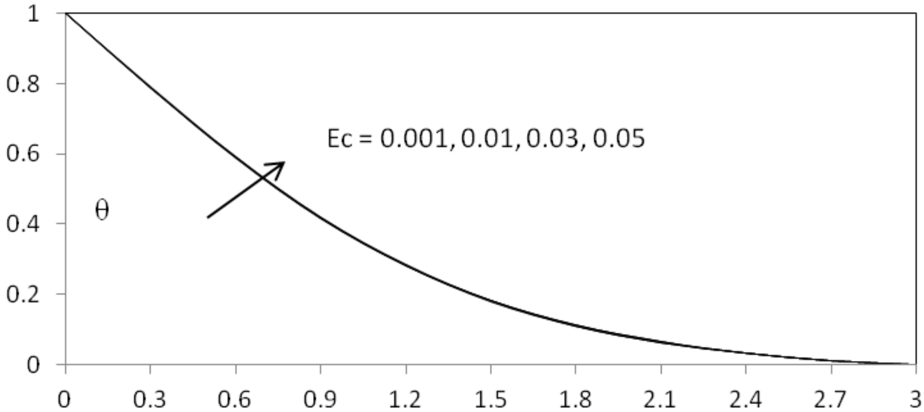

Figure 11: temperature profile with Ec.

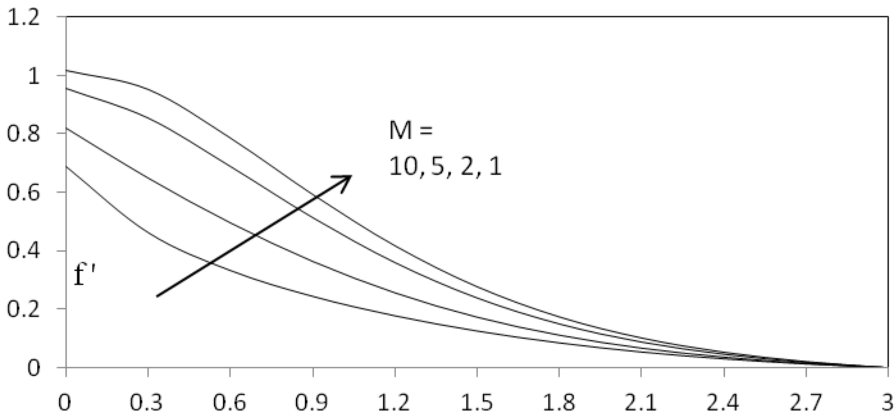

Figure 12: velocity profile with M.

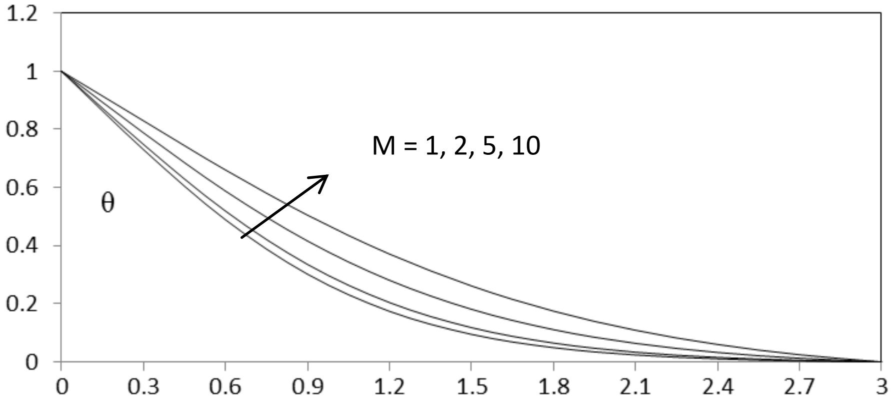

Figure 13: temperature profile with M

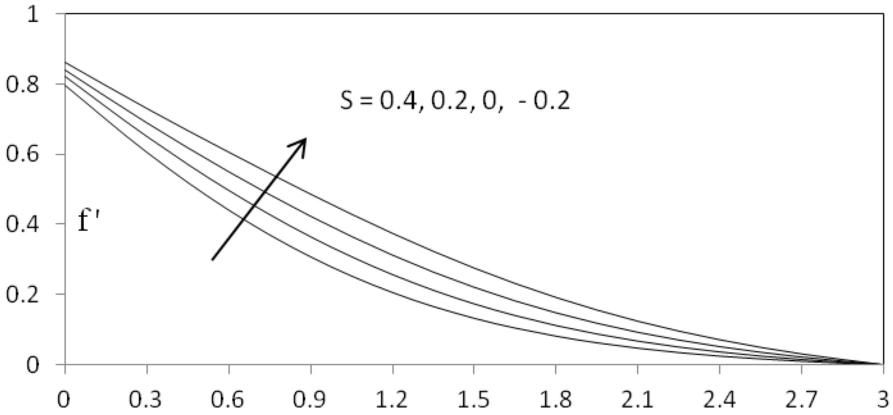

Figure 14: velocity profile with S.

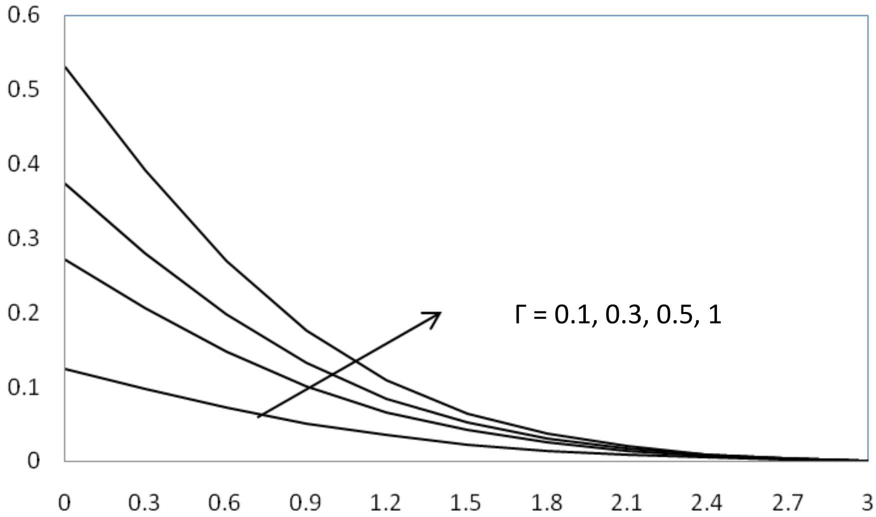

Figure 15: temperature profile with Γ

Table 2: Nusselt Number and Skin Friction proportionate values

Q	Ec	φ	Present Work		Bhaskar Reddy et.al.	
			- f''	- θ'	- f''	- θ'
0	0	0	1.17786	0.095394	1.177071	0.095523
0	0	0.05	1.09201	0.095745	1.09394	0.095117
0	0	0.1	1.01357	0.094377	1.013062	0.094707
0	0	0.15	0.933471	0.094603	0.934425	0.094288
0.5	0.01	0.05	0.917017	0.079196	---	---
0.8	0.01	0.05	0.799862	0.040299	---	---
0.5	0.03	0.05	0.915426	0.078647	---	---
0.5	0.05	0.05	0.913844	0.0781	---	---

Table 2 shows the skin friction and Nusselt number values for the variation of parameters. The values are compared with Bhaskar Reddy. N. et.al. [5]. Skin friction decreases with increase in volume fraction of the Cu particles. Rate of heat transfer gradually decreases with increase in volume fraction. This is due to the high conductivity of the metal particles at higher concentrations. Thus we can say that the conductive heat transfer dominates the convective heat transfer. The increase in heat source reduces the skin friction slightly but reduces the rate of heat transfer significantly. Thermally conductive Cu particles hold the heat energy given to the system so that the heat transfer reduces with heat source. The increase in viscous dissipation reduces skin friction and rate of heat transfer slightly.

Conclusion:

- ✓ The radiative heat transfer is more significant than conduction.
- ✓ The convection and suction enhances the temperature and momentum respectively.
- ✓ The heat transfer rate decreases with solid volume fraction due to Cu metal particles.

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References:

1. S. Choi, "Enhancing thermal conductivity of fluids with nanoparticle," in Developments and Applications of Non-Newtonian Flows, D. A. Siginer and P. H. Wang, Eds., vol. 66, pp. 99–105, ASME, New York, NY, USA, 1995.
2. Yao, S.; Fang, T.; Zhong, Y. Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. Commun. Nonlinear Sci. Numer. Simul. 2011, 16, 752– 760.
3. Rahman, M. Locally similar solutions for hydromagnetic and thermal slip flow boundary layers over a flat plate with variable fluid properties and convective surface boundary condition. Meccanica 2011, 46, 1127–1143.
4. GVPN Srikanth, G Srinivas, B Tulasi Lakshmi Devi, B Suresh Babu "Nano-Particle Size and Inter Particle Spacing Effects on Convective Heat And Mass Transfer Past A Permeable Inclined Oscillating Stretching Sheet" Global Academic Research Journal vol – III, Issue – I, pp 26-34, 2015.
5. N. Bhaskar Reddy,1 T. Poornima,1 and P. Sreenivasulu "Influence of Variable Thermal Conductivity on MHD Boundary Layer Slip Flow of Ethylene-Glycol Based Cu Nanofluids over a Stretching Sheet with Convective Boundary Condition" International Journal of Engineering Mathematics, vol 2014, Article ID 905158.
6. Koo, J.; Kleinstreuer, C. Viscous dissipation effects in microtubes and microchannels. Int. J. Heat Mass Transf. 2004, 47, 3159–3169.
7. M. Arunachalam and N. R. Rajappa, "Forced convection in liquid metals with variable thermal conductivity and capacity," Acta Mechanica, vol. 31, no. 1-2, pp. 25–31, 1978.
8. T. C. Chiam, "Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet," Acta Mechanica, vol. 129, no. 1-2, pp. 63–72, 1998.
9. L. J. Crane, "Flow past a stretching plate," Zeitschrift f¨ur angewandte Mathematik und Physik, vol. 21,no. 4, pp. 645–647, 1970.
10. B. K.Dutta, P. Roy, and A. S. Gupta, "Temperature field in flow over a stretching sheet with uniform heat flux," International Communications in Heat and Mass Transfer, vol. 12, no. 1, pp.89–94, 1985.
11. C. K. Chen and M. I. Char, "Heat transfer of a continuous, stretching surface with suction or blowing," Journal of Mathematical Analysis and Applications, vol. 135, no. 2, pp. 568–580, 1988.
12. W. A. Khan and I. Pop, "Boundary-layer flow of a nanofluid past a stretching sheet," International Journal of Heat and Mass Transfer, vol. 53, no. 11-12, pp. 2477–2483, 2010.
13. M. Hassani, M. M. Tabar, H. Nemati, G. Domairry, and F. Noori, "An analytical solution for boundary layer flow of a nanofluid past a stretching sheet," International Journal of Thermal Sciences, vol. 50, no. 11, pp. 2256–2263, 2011.
14. P. Rana and R. Bhargava, "Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: a numerical study," Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 1, pp. 212–226, 2012.
15. M.A.Hamad and M. Ferdows, "Similarity solution of boundary layer stagnation-point flow towards a heated porous stretching sheet saturated with a nanofluid with heat absorption/generation and suction/blowing: a Lie group analysis," Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 1, pp. 132–140, 2012.
16. M. A. Seddeek and A. M. Salem, "Laminar mixed convection adjacent to vertical continuously stretching sheets with variable viscosity and variable thermal diffusivity," Heat and Mass Transfer/Waerme- und Stoffuebertragung, vol. 41, no. 12, pp.1048–1055, 2005.
17. Vajravelu, K., and Hadjinicolaou, A., (1993), Int. Comm. Heat Mass Trans., Vol.20, pp.417-430.

18. Molla, M. M., Hossain, M. A., and Yao, L. S., (2004), Int. J. Thermal Sci., Vol.43, pp.157-163.
19. Samad, M. A., and Mohebujjaman, M., (2009), Research J. of Applied Sci., Eng. and Tech., Vol.1 (3), pp.98-106.
20. Van Rij, J.; Ameel, T.; Harman, T., The effect of viscous dissipation and rarefaction on rectangular microchannel Convective heat transfer. Int. J. Therm. Sci. 2009, 48, 271–281.